A CP-PRESERVING ANALYSIS OF NEUTRAL K-MESON DECAY

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By

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CERTIFICATE

Certified that the work presented in this thesis entitled 'CP-Preserving Analysis of Neutral K-Meson Decay' by Mr. Vijay Kumar Agarwal has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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Page 16 — Second Paragraph, 7^{th} line should read:

" $m_L - \frac{i}{2}\Gamma_L$ in $S_-(E)$ ......

Page 24 — eq. (3) should read:

$$\xi_S / \Gamma_S = \xi_L / \Gamma_L \qquad (3)$$

Page 32 — second line should read:

" written as a sum of two terms, viz., (only for $0 < |\theta|$

— eq (11a) second line should read:

$$- e^{\mp i\beta} (\omega^{*2} - \xi^2)\, e^{\omega^* t} \int_0^t dk\, e^{-\omega^* k} K_0(\xi k)$$

Page 68 — 4^{th} line should read:

$$g(0,\omega) = i\pi / \sqrt{\omega^2 - \xi^2} \quad \text{if} \quad \underline{0 < |\theta| \lesssim \xi}$$

— 7^{th} line should read:

" <u>only if $0 < \xi < T_{1/2}$ and $0 < |\theta| \lesssim \xi$</u> ...

ACKNOWLEDGEMENTS

At the outset, I must express my great indebtedness to Professor Gyan Mohan for guiding me throughout the present investigation with great patience despite my many blunders. His continual encouragement and ready help when needed were invaluable to me.

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V.K. Agarwal

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Thesis entitled 'A CP-Preserving Analysis of Neutral K-meson Decay' submitted by Vijay Kumar Agarwal in partial fulfilment of the requirement of the Ph.D. degree to the Department of Physics, Indian Institute of Technology, Kanpur-208016

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It is universally accepted that the phenomenon of neutral K-decay is to be understood in the context of violations of CP-invariance. However, it appears strange that even after a decade no other phenomenon has been discovered which also requires violation of CP-invariance. A basic principle being so selective in its display is an uncomfortable feature. Consequently, it appears useful to renew an effort to understand the neutral K-decay phenomenon within the requirements of CP-conservation.

Present analysis is made in terms of the S-matrix. The decaying state, in general, is characterized by a pole in the S-matrix. However, it is found that the CP-preserving analysis necessitates more complicated singularity structure of the S-matrix than a simple pole. In the neighbourhood of the pole, a branch point (distinct from the threshold branch points) is postulated. The occurrence of this branch point does not violate causality or unitarity or CP-invariance. Proximity of the branch point to the pole allows for a

leakage resonance- like behaviour in the next sheet round the branch point. Such a situation will lead to two effective resonances in mutually orthogonal channels arising from a single pole. The occurrence of CP = + 1 component of K_L can, therefore, be interpreted as the leakage from a pole in the CP = -1 sheet. The state $|L+\rangle$, corresponding to this leakage term, is produced solely due to the weak-interaction.

The time-dependence of the decay-amplitude with the S-matrix possessing a combination of pole and a branch point is, in general, expected to be non-exponential. However, it is found that for some particular positions of the branch point, the decay-amplitude can be fairly well represented by an exponential function. Further, the parameters of the two pion interference in kaon decay determine the actual position of the branch point.

The existing experimental data are all shown to be consistent with the proposed scheme. However, under suitable experimental set up the unusual behaviour of the leakage state, $|L+\rangle$, does lead to clear distinctions of the proposed scheme with the conventional CP-violating (CPV) theories. Whenever, the beam of delayed kaon passes through a nuclear material, the state $|L+\rangle$ emerges largely unaffected while $|L-\rangle$ (corresponding to CP = -1 with pole at M_L) is attenuated. Therefore, the asymptotic

value of the leptonic charge-asymmetry following regeneration is expected to be dependent upon the thickness of the regenerator in contrast with the results of the CPV-phenomenology where this value is same for all regenerators. Furthermore, the two pion interference is expected to exhibit more complex dependence on thickness of regenerator in the proposed analysis than in the CPV-analysis. The existing experimental data are indecisive about such features because of large experimental errors.

CHAPTER I

INTRODUCTION

The neutral K-meson is the only known example of a self conjugate system where one can define any CP-invariance selection rules. Gellmann-Pais[1] first obtained such selection rules by proposing their famous particle-mixture hypothesis and concluded that the CP-eigenstates of neutral K-meson with eigenvalues +1 and -1 decay with different life-times. The one with CP = +1 decays much faster than the CP = -1 component. The discovery of non-conservation of parity[2] touched off subsequent speculations about the possible CP non-invariance. It was suggested by Lee[3] that a possible CP non-invariance in Gellmann-Pais (G-P) scheme will mean existence of delayed two pion (CP = +1) decays. This inturn will lead to the phenomenon of interference between the amplitudes for prompt and delayed two pion decays in a beam of neutral kaon. It was also pointed out that such deviations from G-P scheme will imply a favour to a particular leptonic charge in the semi-leptonic decays.

In 1964, Christensen et al [4] discovered the two pion decay delayed mode. The following year, Fitch et al[5] observed interference in the two pion decays in a beam of neutral kaon. The CP-violation parameter, η, defined by the ratio of

the amplitude for delayed 2 pion decay to the amplitude for prompt two pion decays was measured along with the mass differences of the two kaon states. Later on, the charge-asymmetry in semi-leptonic decays of delayed Kaon was also measured by Dorfon[6] and Bannett[7]. The magnitude of these two parameters was found to be of the order of 10^{-3}. The smallness of the effect suggests that the CP-invariance is otherwise a good symmetry.

There are many theoretical models[8] which were proposed to understand the mechanism of the decay phenomenon of the neutral kaon. The most successful of them all is the superweak model[9]. In this model an interaction with a coupling constant 10^{-9} times the usual weak interaction coupling constant is assumed. This new interaction violates CP-invariance and admits $|\Delta S| = 2$ transitions, namely $K_o \leftrightarrow \bar{K}_o$. The two pion decay interference-phenomenon can be easily demonstrated and the phase of the parameter η is given by,

implied by $\Delta m \sim \Gamma_S/2$ (to order g')

$$\text{phase } \eta = \tan^{-1} (2\Delta m/\Gamma_S) \simeq \pi/4 \qquad ; \ \Delta m \sim \frac{g'}{m_\pi^{-3}} \qquad g' = G^2 m_\pi^2$$

The subsequent experiments, surprisingly, lead to a phase as predicted by superweak theory[10].

The phenomenological models proposed so far to describe the behaviour of the neutral kaon, are all based on the assumptions of a simple time-dependence for the

decay amplitude. This time-dependence is more conveniently understood in terms of a singularity of the appropriate S-matrix[11]. The normal analyticity postulate[12] associates a resonance state with an isolated pole in the S-matrix. However, this postulate is not well-established[13]. In fact, there exists at least one class of field theories[14] where the S-matrix is not analytic in conventional sense.

We would like to explore the possibility that the neutral kaon resonances are represented by more complicated singularity structures than simple isolated poles. The new structures will include branch points which, however, do not violate causality or unitarity. In the work presented here, we investigate a model proposed earlier[15] to understand the neutral K-meson phenomena within CP-preserving framework. It was the conclusion of reference 15 that such phenomenology forces inclusion of branch points (distinct from the threshold branch points) besides the usual pole in the S-matrix.

In Chapter III of the present work, we calculate the time-evolution of the neutral K-meson state in terms of the proposed S-matrix. It is found that the time-dependence of the decay-amplitude is, in general, non-exponential. However, for some particular positions of the branch point, the decay amplitude can be fairly well approximated by an exponential term. The equality of the parameter η, for charged pion and

and neutral pion decays of neutral kaon, is found to be trivial because of the occurrence of the same S-matrix element in the corresponding amplitudes. The phase of the parameter η fixes the position of the branch point.

In Chapter IV, the regeneration in a nuclear material is considered and clear distinctions from the conventional CP-violating (CPV) approach brought about. Since the multiple scatterings in a thick regenerator contribute significantly, these effects are also included in the calculations.

In Chapter V, the two important phenomena namely, the two pion interference and charge-asymmetry in semi-leptonic decays, are considered in detail. The calculations are compared with the CPV-results and also the experimental data. It is found that the present experimental data can be understood within the phenomenology, however, more critical and decisive experiments need to be done. Some probable future experiments to test the proposed analysis are suggested in the concluding remarks.

CHAPTER II

NEUTRAL K-MESON STATE AND THE S-MATRIX

The phenomenological neutral K-meson state describing the decay phenomena is considered as a resonance in an appropriate scattering experiment. The details of the scattering experiment are not relevent to the subsequent analysis which is performed in terms of the out-states. However, the separation of the resonance from other fragments of the scattering experiment will require care. As an illustration, let us consider a broad featureless superposition over energies of the in-states:

$$|\psi\rangle = N \sum_{\alpha} \int dE\, a_{\alpha}(E)|\, E,\, a;\, in\rangle \qquad (1)$$

To study the subsequent decay properties, we must analyse (1) in terms of out-states i.e.

$$|\psi\rangle = N \sum_{\alpha,\beta} \int dE|\, E,\, \beta;\, out\rangle \; S_{\beta\alpha}(E)\; a_{\alpha}(E) \qquad (2)$$

The superposition coefficient S(E) a(E) in (2) may or may not have sharp energy dependence. The time-dependence of (2) is given by,

$$|\psi\, ;\, t\rangle = N \sum_{\alpha,\beta} \int dE\, e^{-iEt}|\, E,\, \beta;\, out\rangle\, S_{\beta\alpha}(E)\; a_{\alpha}(E) \qquad (3)$$

Now if S(E) is broad and featureless i.e. smooth function of E, the relative phases of $|E, \beta; \text{out}>$ in $|\psi; t>$ will change rather quickly with time because a wide range of values of E contribute significantly to (3). On the otherhand, if S(E) is sharply peaked in E, then for all the practical purposes the relative phases in $|\psi; t>$ would change significantly only in periods larger than the order of inverse of peak-width. But unitarity implies that S(E) is essentially a phase and hence there cannot be peaking, consequently (3) must describe a state changing rapidly with time. This conclusion is not necessarily true because [S(E)-1] could be a peaked function. This will mean, the resonance will show if properly separated from the unscattered amplitude. If only we wait for sufficient time, the resonance part can be filtered. The resonance state can be written as:

$$|\psi_r> = N \sum_{\alpha,\beta} \int dE |E, \beta; \text{out}> [S_{\beta\alpha}(E) - \delta_{\beta\alpha}] \, a_\alpha(E) \quad (4)$$

A simple pole in the S-matrix means a simple pole in the determinant of the S-matrix. Weidenmuller[16] and Goebel and McVoy[17] (also McVoy[18]) have noted that the eigenvalue functions of the S-matrix, in general, possess complicated analytic properties even when the S-matrix has a simple pole. One expects a single multiple-valued analytic function to represent, in various sheets, all the eigenvalue

functions. If the branch point is near the pole, then as one scans through the energy values over a range clear across the resonance, a sequential fast rise of the successive eigenphases (connected at this branchpoint) ensues such that the sum of the eigenphases increase through π. Thus the single pole, which occurs in just one of the eigenvalue-function sheet, activates all the phases and no one of them will show a pure exponential decay. The non-exponential nature of the decay arises from the deficit in reaching the normal resonance form for the phases. However, if the branch point is far away, or if there are special circumstances leading to a cancellation of the branchpoints, one has an isolated pole in a single eigenvalue-function which does not activates any other eigenvalue-function and hence displays exponential decay characteristics.

2.2 THE PHENOMENOLOGICAL K_0-STATE VECTOR:

The phenomenological state vector for neutral kaon is constructed as a coherent superposition of all its decay channels. The time-dependence of this state is given in terms of two S-matrix functions, $S_+(E)$ and $S_-(E)$ corresponding to CP-eigenvalues +1 and -1 respectively. We also assume that the phase space factors have been absorbed in the superposition coefficients. Furthermore, it is assumed that the same S-matrix function with a resonance form describes all the

decay channels corresponding to one CP-eigenvalue. The superposition coefficients are, in general, smooth functions of E, so that for all practical purposes these coefficients may be replaced by constants. Therefore, the neutral kaon state tentatively is written as

$$| K_o; t > = N \int dE\, e^{-iEt} [S_+(E)-1] \sum_m C_m^+ |E,m,+; \text{out} >$$
$$+ N \int dE\, e^{-iEt} [S_-(E)-1] \sum_n C_n^- | E,n,-; \text{out} > \qquad (5)$$

where +ve and -ve sign represent the CP-eigenvalue and m or n collectively stand for various quantum-numbers characterizing the various decay channels.

The various partial decay-widths will determine the magnitude of the coefficients C's. It can also be noticed[19] that $\Delta I = 1/2$ rule will suppress the coefficients $C^+_{2\pi,I=2}$, $C^+_{3\pi,I=2}$ and $C^-_{3\pi,I=3}$. The coefficients $C^+_{3\pi,I=0}$ and $C^+_{3\pi,I=2}$ will be suppressed by the centrifugal barrier. The three particle states require further characterization regarding the mode of energy sharing.

The phases of the superposition coefficients are fixed by Watson's theorem[20] which gives the phases in terms of the asymptotic situation where the de cay-interaction is absent. That means the phases of $C_{2\pi}$ and $C_{3\pi}$ are given in terms of the corresponding strong-interaction phaseshifts and the phases for $C^+_{\pi l\nu}$ and $C^-_{\pi l\nu}$ are equal for same helicity of lepton.

The normalization of (5) together with the preservation of CP, implies equal weights to both the CP-sectors, i.e.

$$\frac{1}{2} = 4N^2 \int dE \, \sin^2 \delta_+(E) \, \sum_m |C_m^+|^2$$

$$= 4N^2 \int dE \, \sin^2 \delta_-(E) \, \sum_n |C_n^-|^2$$

where,

$$S_\pm(E) = \exp\left[2i\,\delta_\pm(E)\right]$$

The probability amplitude that the state (5) at any time t is still the state $|K_o\rangle$, is given by the composite part of

$$\langle K_o | K_o ; t \rangle = 4N^2 \sum_m \int dE \, \sin^2 \delta_+(E) \, e^{iEt} \, |C_m^+|^2$$

$$+ 4N^2 \sum_n \int dE \, |C_n^-|^2 \, e^{-iEt} \sin^2 \delta_-(E) \qquad (6)$$

This will indicate a sustained amplitude if $\sin^2 \delta(E)$ has a sharp peak. A typical form arising from an isolated pole in the S-matrix,

$$\sin^2 \delta(E) = (\Gamma/2)^2/[(E-E_o)^2 + \Gamma^2/4]$$

leads to an amplitude

$$\int dE \, e^{-iEt} \sin^2 \delta(E) = \frac{\pi\Gamma}{2} \, e^{-iE_o t - \Gamma t/2}$$

However, ours is not such a simple case. The experimental observation that the long lived component of neutral K-meson also decays to CP = +1 eigenchannel, implies that the structure of the S-matrix function $S_+(E)$ should be such that the amplitude

$$\int dE\ e^{-iEt} \sin^2 \delta_+(E)$$

clearly reveals two life-times, corresponding to the prompt (K_S) and the delayed (K_L) components.

2.3 MODEL WITH OVER-LAPPING RESONONANCES:

A simple and crude model to describe the neutral K-meson phenomena can be given by assuming the CP = +1 amplitude arising from two overlapping-simple resonances. The phaseshift $\delta_+(E)$ is, then, expressed as an appropriate addition of two phaseshifts $\delta_S(E)$ and $\delta_L(E)$, each representing a simple isolated resonance at masses and widths corresponding to K_S and K_L respectively. If now we consider the trivial identity,

$$e^{2i\delta_+(E)} = e^{2i\delta_S(E)} + e^{2i\delta_S(E)} [e^{2i\delta_L(E)} - 1] \quad (7)$$

then, since the function $[e^{2i\delta_L(E)} -1]$ is sharply peaked at $E = m_L$ and because $(\Gamma_L/\Gamma_S) << 1$, the phaseshift $\delta_S(E)$ does not change appreciably in the neighbourhood of $E = m_L$ within a span of a few Γ_L; the expression (7) can be approximated by

$$e^{2i\delta_+(E)} \simeq e^{2i\delta_S(E)} + e^{2i\delta_S(m_L)} [e^{2i\delta_L(E)} -1] \quad (8)$$

The use of the approximation (8) requires care, because of the unitarity of the analysis. Therefore, in any calculation the approximation (8) is substituted as the last step (before actually carrying out the integrations). The expression (6) will, then, lead to,

$$\int dE\ e^{-iEt}\ \sin^2\ \delta_+(E) \simeq \int dE\ e^{-iEt}\ \sin^2\ \delta_S(E) + e^{2i\ \delta_S(m_L)} \int dE\ e^{-iEt}\ \sin^2\ \delta_L(E) + \text{transient term}$$

which clearly reveals two life-times.

The parameter η, (i.e. the ratio of the amplitudes for decays $K_L \to 2\pi$ to $K_S \to 2\pi$) is

$$\eta = (\Gamma_L/\Gamma_S)\ e^{2i\delta_S(m_L)}$$

which agrees with the data in magnitude but the phase is much larger than the experimental value.

2.4 BRANCH POINTS IN S-MATRIX:

The failure of the simple overlapping-resonance model suggests that the rate of increase of the phaseshift should be arrested by some appropriate mechanism. A simple way to achieve it is introduction of branchpoints in the vicinity of the pole in the S-matrix[15,21]. These branchpoints do not violate causality or unitarily (which demands these branch points occur on conjugate positions in complex E-plane).

The occurrence of these branch points at complex energies is a rather unusual phenomena. The conventional analyticity postulate[12] of the S-matrix theory does not permit such branch points. However, the analyticity postulate as such is not well-established[13] and there does exist at least one class of representations of the S-matrix[14], which violate the analyticity postulate. Therefore, such occurrences of branch points in the S-matrix can not be lightly disregarded unless experiments clearly disprove their existence. These branch points are part of dynamics and their position should be determined by the specific interaction.

A simple description incorporating these branch points in the S-matrix can be given in terms of the eigenvalue functions of the S-matrix.[16,17] Each eigenvalue function can be considered as a branch of an analytic function of E. This analytic function of E is chosen to have two Riemann sheets (Figure 1(a)). On any one of these Riemann sheets <u>any 'physical' point</u> (corresponding to real positive energies) <u>can not be connected to any 'physical' point on the other sheet by any simple transformation</u> in the E-plane i.e. the two sheets are unconnected for physical energies. This 'physical-unconnectedness' of the two sheets is a consequence of the fact that the two eigenvalue functions correspond to orthogonal channels.

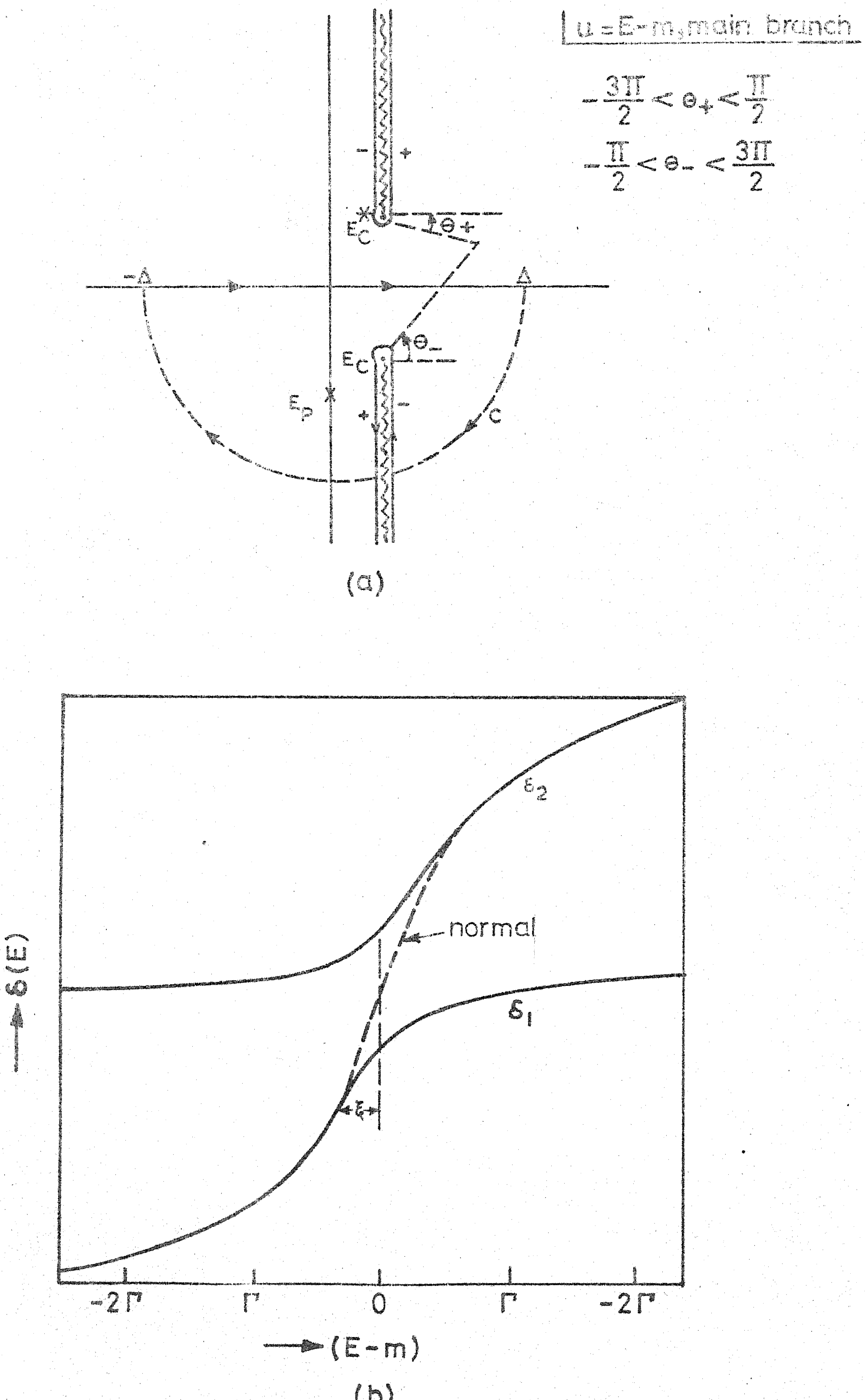

$u = E - m$, main branch
$-\frac{3\pi}{2} < \theta_+ < \frac{\pi}{2}$
$-\frac{\pi}{2} < \theta_- < \frac{3\pi}{2}$
E_C
θ_+
$-\Delta$
Δ
θ_-
E_C
E_p
C
(a)
δ_2
normal
δ_1
ξ
$\delta(E)$
-2Γ
Γ
0
Γ
-2Γ
(E - m)
(b)

Figure. 1

A simple pole in one of the sheets will induce a resonance-like behaviour for the eigenvalue function on the other sheet corresponding to an orthogonal channel. An interesting consequence of this 'leakage' of the pole is that the main eigen phase (the one carrying the pole) does not increase through π (rapidly) as is expected in the usual resonance behaviour of the phase shifts, but reaches a stagnation value much earlier (around the position of the branch point) and then varies slowly characteristic of non-resonant behaviour (Figure 1(b)). The two eigenphases exhibit a remarkable energy dependence near the position of the branch point. It is seen[17,18] that the two eigenphases never cross each other but repel[22] such that the total increase in the phase (for the eigenphases) is π when one crosses the pole. The energy band over which such a repulsion (which also causes the stagnation of the main phase) occurs is given in terms of the imaginary part of the position of the branch point (denoted by ξ).

For $\xi \geq \varepsilon$ for any $\varepsilon > 0$, the two Riemann sheets are completely specified in a consistent way. However, for $\underline{\xi = 0}$ the basic structure changes. Since the pair of branch points collide on the real E-axis, the two Riemann sheets have a very different structure. On the same sheet, the entire regions on the left and on the right side of the cut are specified by two completely different functions.

This situation, therefore, represents a 'discontinuity' in the conceptual sense arising from the inadequacy of the complete specification of the Riemann sheets by physical description. However, this difficulty can be avoided by conjecturing that ξ does not take the value 0 for all the processes of interest.

For very large ξ, on the otherhand, the main eigenphase has sufficient room to increase through π and therefore, represent an isolated resonance, while the 'leakage' eigenphase will be dormat and inconsequential. The deviations of the phase shift from the normal resonance shape will manifest themselves in terms of non-exponential behaviour of the associated time-dependent decay amplitude. The extent of these deviations from the exponential decay law will, of course, depend upon the position of the branch points, specially on the quantity ξ. However, it is obvious that for very large ξ (greater than $\Gamma/2$) the non-exponential terms will not be important at least for the times of the order of tens of the life-time.

2.5 THE CP-PRESERVING MODEL FOR NEUTRAL KAON PHENOMENA:

We propose to incorporate the m_L resonance corresponding to CP = +1 eigenvalue as a leakage resonance from the pole at $(m_L - \frac{i}{2}\Gamma_L)$ in $S_-(E)$ sheet. This will not only ensure approximate equality of masses and life-times for

the two delayed modes, but also preserve the CP-invariance. The structure of the Riemann sheets is given by,

(a) a pole near $m_L - \frac{i}{2}\Gamma_L$ in $S_-(E)$ sheet,

(b) a branch point very near to this pole such that round the branch point lies the $S_+(E)$ sheet,

(c) a pole near $m_S - \frac{i}{2}\Gamma_S$ in the $S_+(E)$ sheet, and

(d) a branch point in $S_+(E)$ near this pole, which connects it to a different sheet. (We could, however, consider this other sheet to be $S_-(E)$, but only for simplicity we would not).

The phaseshift $\delta_+(E)$ will show two distinct sharp rises, although not quite overlapping. We can again write

$$\delta_+(E) = \delta_S(E) + \delta_L(E)$$

where the phaseshifts $\delta_S(E)$ corresponds to the S-matrix eigenvalue function carrying the pole at $m_S - \frac{i}{2}\Gamma_S$, and the phaseshift $\delta_L(E)$ corresponds to the leakage of the pole at $m_L - \frac{i}{2}\Gamma_L$ in $S_-(E)$. Because of the branch point the eigenphase $\delta_S(E)$ does not rise through π (as is normally the case) but stagnates to some value α_S. If $\delta_S^t(E)$ be the non-resonant part of $\delta_S(E)$ for sub-resonant energies i.e. $[\delta_S(E) - \delta_S^t(E)]$ is negligible for $E-m_S \geq -\Gamma_S$, then $[\delta_S(E)-\delta_S^t(E)-\alpha_S]$ is negligible for $E-m_S \geq \Gamma_S$. Similarly, let the non-resonant part of $\delta_L(E)$ be $\delta_L^t(E)$ for sub-resonant energies. A decomposition of $S_+(E)$ similar to (7) may be

obtained via the trivial identity:

$$S_+(E) = e^{2i\delta_S(E)} e^{2i\delta_L^t(E)} + e^{2i\delta_S(E)} [e^{2i\delta_L(E)} - e^{2i\delta_L^t(E)}].$$

The approximation (8) can still be carried if the phaseshift $\delta_S(E)$ stagnates earlier than $E = m_L$. This will put stringent conditions on the parameter ξ, specifying the position of the branch point. If such be the case, then we can write,

$$S_+(E) \simeq e^{2i\delta_S(E)} e^{2i\delta_L^t(E)} + e^{2i\alpha_S} e^{2i\delta_S^t(E)+2i\delta_L^t(E)} [e^{2i[\delta_L(E)-\delta_L^t(E)]} - 1] \quad (9)$$

The CP = +1 part of the amplitude will again contribute two time dependences.

The rather large background phaseshifts need to be removed carefully in the definition of K_o-state vector, otherwise it will give rise to a large transient term. Therefore, we will replace $[S_+(E)-1]$ by $[S_+(E)-\exp 2i[\delta_S^t(E) + \delta_L^t(E)]]$ and hence by $[\exp -2i[\delta_S^t(E) + \delta_L^t(E)].S_+(E)-1]$ and thus remove the low energy non-resonant amplitude. The high energy non-resonant amplitude is removed by truncating the range of E-integration just beyond the two resonances. The resultant transient term is carefully removed. This process corresponds to considering just the composite part of the amplitudes.

(a) The K_o-State:

The neutral K-meson state according to the definition (5) and the decomposition (9) can be written as,

$$| K_o > \; = \; b | L- > + a_S | S+ > + a_L | L+ > \tag{10}$$

where,

$$b | L- > = N \sum_n C_n^- \int dE \; (e^{2i[\delta_- - \delta_-^t]} - 1) | E,n,-;out> \tag{11a}$$

$$a_S | S+ > = N \sum_m C_m^+ \int dE \; (e^{2i[\delta_S - \delta_S^t]} - 1) | E,m,+;out> \tag{11b}$$

and

$$a_L | L+ > = e^{2i\alpha_S} N \sum_m C_m^+ \int dE \; (e^{2i[\delta_L - \delta_L^t]} - 1) | E,m,+;out > \tag{11c}$$

The states $|L- >$, $|S+ >$ and $|L+ >$ are linearly independent states where S and L stand for short- and long-life-time and + and - refer to the corresponding CP-eigenvalue. The state $|L+ >$ is, by definition, due to the leakage of the pole at $m_L - \frac{i}{2}\Gamma_L$ in the CP = -1 eigenchannel. These states are normalized according to the relations:

$$\begin{aligned} < L- | L- > \; &= \; 1 \; = \; < S+ | S+ > \; = \; < L+ | L+ > \\ < L- | S+ > \; &= \; 0 \; = \; < L- | L+ > \\ < L+ | S+ > \; &\neq \; 0 \end{aligned} \tag{12}$$

The coefficients a_S, a_L and b can be determined by the normalization conditions. An order of magnitude calculation immediately leads to an estimation:

$$|b| = 1/\sqrt{2} \qquad |a_S| \sim 1/\sqrt{2} \qquad |a_L| \sim (\Gamma_L/\Gamma_S)^{1/2}$$

$$\frac{\sum_m |C_m^+|^2}{\sum_n |C_n^-|^2} \sim (\Gamma_L/\Gamma_S) .$$

The phase of a_L can be obtained by the expression,

$$a_S^* \, a_L \langle S+|L+\rangle \simeq N^2 \sum_m |C_m^+|^2 . \, [1-e^{2i\alpha_S}]$$

$$\int dE \, (e^{2i[\delta_L-\delta_L^t]} -1) \tag{13}$$

The phases of a_S and b can not be determined by these normalization relations and therefore, these can be chosen real.

The state $|K_o\rangle$ can be conveniently written in terms of the parameter, ε_1, i.e.

$$|K_o\rangle = \frac{1}{\sqrt{2}} \left[|L-\rangle + \frac{1}{x_1} |S+\rangle + \frac{\varepsilon_1}{x_1} |L+\rangle \right] \tag{14}$$

where

$$\varepsilon_1 = a_L/a_S \quad \text{and} \quad x_1 = 1+|\varepsilon_1|^2 + 2\langle S+|L+\rangle \, \mathrm{Re}\,\varepsilon_1 .$$

(b) $\overline{K}_o$-State:

The construction of the phenomenological $|\overline{K}_o>$ requires some care because the particle and antiparticle relations between K_o and $\overline{K}_o$ is defined only in the absence of the decay interaction. The state $|\overline{K}_o>$ is constructed in the following manner: we first construct the $|\overline{K}_o>$ state in the limit of strong interaction as the usual antiparticle state orthogonal to the state $|K_o>$. The finer structures to the state are introduced by switching on the weak interaction (which leads to the leakage phenomena). That is, the $|\overline{K}_o>$ state is constructed by incorporating leakage phenomena to the state $[|L->-|S+>]$, where each component is represented by a corresponding isolated pole. Therefore, we will have,

$$|\overline{K}_o> \;=\; b\,|L-> + a_L'\,|L+> - a_S'\,|S+> \qquad (15a)$$

The normalization of (15a) will lead to

$$|b| = 1/\sqrt{2} \quad \text{and} \quad |a_L'|^2 + |a_S'|^2 - 2\mathrm{Re}(a_S'\,a_L')<L+|S+> = \frac{1}{2}$$

We can rewrite (15a) as,

$$|\overline{K}_o> = \frac{1}{\sqrt{2}}\,[|L-> + \frac{\varepsilon_2}{x_2}|L+> - \frac{1}{x_2}|S+>] \qquad (15b)$$

where,

$$\varepsilon_2 = \frac{a_L'}{a_S'} \quad \text{and} \quad x_2 = 1 + |\varepsilon_2|^2 - 2\mathrm{Re}\;\varepsilon_2<S+|L+>$$

The parameters ε_1 (occurring in the definition of K_o) and ε_2 can be determined by experiments by either studying the time-dependent decay intensities for pure K_o and pure $\bar{K}_o$ separately or by a best fit to the intensities for a mixed beam of K_o and $\bar{K}_o$. The explicit fiting is carried in Chapter V.

(c) Weak-Interaction Resonance:

In the space of three linearly independent state-vectors $|L-\rangle$, $|S+\rangle$ and $|L+\rangle$, we can construct another set of three linearly independent state-vectors, whose first two members are identified to be the usual $|K_o\rangle$ and $|\bar{K}_o\rangle$ and the third can be denoted by $|K'\rangle$. The new state $|K'\rangle$ can be written as,

$$|K'\rangle = C_S|S+\rangle + C_L|L+\rangle \tag{16}$$

such that

$$\begin{aligned} \langle K_o|K'\rangle &= 0 = \langle \bar{K}_o|K'\rangle \\ \langle K'|K'\rangle &= 1 \end{aligned} \tag{17}$$

An order of magnitude calculation leads to an estimation,

$$|C_S| \sim (\Gamma_L/\Gamma_S)^{1/2} \quad \text{and} \quad |C_L| \sim 1$$

That means, the state $|K'\rangle$ is largely due to the leakage contribution i.e. dominated by $|L+\rangle$. Therefore, K' is very distinct from the usual neutral kaon states.

If the branch point moves farther off the pole, there is no effective leakage of the pole at $m_L - \frac{i}{2}\Gamma_L$ and the amplitude for the state $|K'>$ becomes indistinguishable from the transient terms. $|K'>$, therefore, acquires an increasingly diffuse energy spread and short life-time. We believe that the location of the branch point is very sensitive to the weak interaction in absence of which K' would become undetectable. In contradistinction, in the absence of weak interaction the states $|K_o>$ and $|\bar{K}_o>$ become absolutely sharp and the states $|L->$ and $|S+>$ are both represented by isolated poles. Thus unlike $|K_o>$ and $|\bar{K}_o>$ the new state $|K'>$ is entirely due to the weak interaction. The strong interaction produces K_o and $\bar{K}_o$ and given enough time, the weak interaction produces K' in the same beam. When such a beam passes through nuclear material, K_o and $\bar{K}_o$ will interact strongly and will, therefore, be absorbed while K' will emerge largely unaffected. The K'-particle was first introduced by Uretsky[23] and Lipkin and Abashian[24] in context of 'CP-violation'.

CHAPTER III

THE S-MATRIX AND TIME-DEPENDENCE OF THE AMPLITUDE

In the previous chapter, the neutral kaon state was constructed in a phenomenology where the CP = +1 component of m_L resonances is interpreted as a leakage of the pole at $m_L - \frac{i}{2}\Gamma_L$ in $S_-(E)$ sheet into $S_+(E)$ sheet. Because of the occurrence of the branch point the eigenphases deviate from the normal resonance shape and consequently the amplitude, in general, is expected to have a non-exponential time-dependence. In this chapter, we will investigate the time-dependence of the amplitude by assuming a simple E-dependence for the S-matrix eigenvalue functions. It is shown, by explicit calculations that the position of the branch point can be so adjusted that not only is there a significant leakage of the pole, but also the deviations from the exponential decay law are exceedingly small. Furthermore, these position of the branch point do lead to values of the interference parameter η consistent with the experiments.

3.2 FORM OF S-MATRIX EIGENVALUE FUNCTIONS:

The unitary function whose branches (σ_+ and σ_-) represent the two S-matrix eigenvalue functions is assumed to have an E-dependence:

$$\sigma_{\pm}(E) = \frac{(E-m)\cos\beta + \Gamma/2\,\gamma\sin\beta \pm i\sin\beta\,|E-E_c|}{E - m + \frac{i\Gamma}{2}} \tag{1}$$

where the positions of the branch points, E_c and E_c^*, are given in terms of the dimensionless parameters (β, γ),

$$\begin{aligned} \theta &\equiv \mathrm{Re}\,(E_c - m) = \Gamma/2\,\gamma\cot\beta \\ \xi &\equiv \mathrm{Im}\,E_c = \Gamma/2\sqrt{1-\gamma^2}\,\mathrm{cosec}\,\beta \end{aligned} \tag{2}$$

The phase shifts, $\delta_{\pm}(E)$, associated with the functions $\sigma_{\pm}(E)$ have the asymptotic values,

$$\beta/2 \leq \delta_-(E) \leq \pi-\beta/2$$

$$\pi-\beta/2 \leq \delta_+(E) \leq \pi+\beta/2$$

and the repulsion band (centered about $E\sim m+\theta$) is given by ξ. Since the same interaction determines the states corresponding to poles at m_S and m_L, we hopefully expect that the relative repulsion bands (i.e. values of ξ) are same i.e. we assume

$$\xi_S/\Gamma_S = \xi_L/\Gamma_L \tag{3}$$

The quantity θ determines the extent of the leakage of the pole e.g. $\theta \sim 0$ will mean the two eigenphases are approximately given by the two halves (about $E = m$) of the normal resonance shape and therefore, will represent the maximum leakage. $\theta \gg \Gamma$ (i.e. corresponding to $\beta \sim 0$) will correspond to the extreme situation when the pole is effectively in the σ_- (E) sheet. The value $\gamma = 1$ (i.e. $\xi = 0$) is not permissible because the Riemann sheets are not properly defined (see section 2.4).

The branch $\sigma_-(E)$, corresponding to the sign of $|E-E_c|$ term to be -ve in (1), has non-vanishing residue at the pole (for $\xi \neq 0$) and is identified as the 'main' branch of the analytic function $\sigma(E)$.

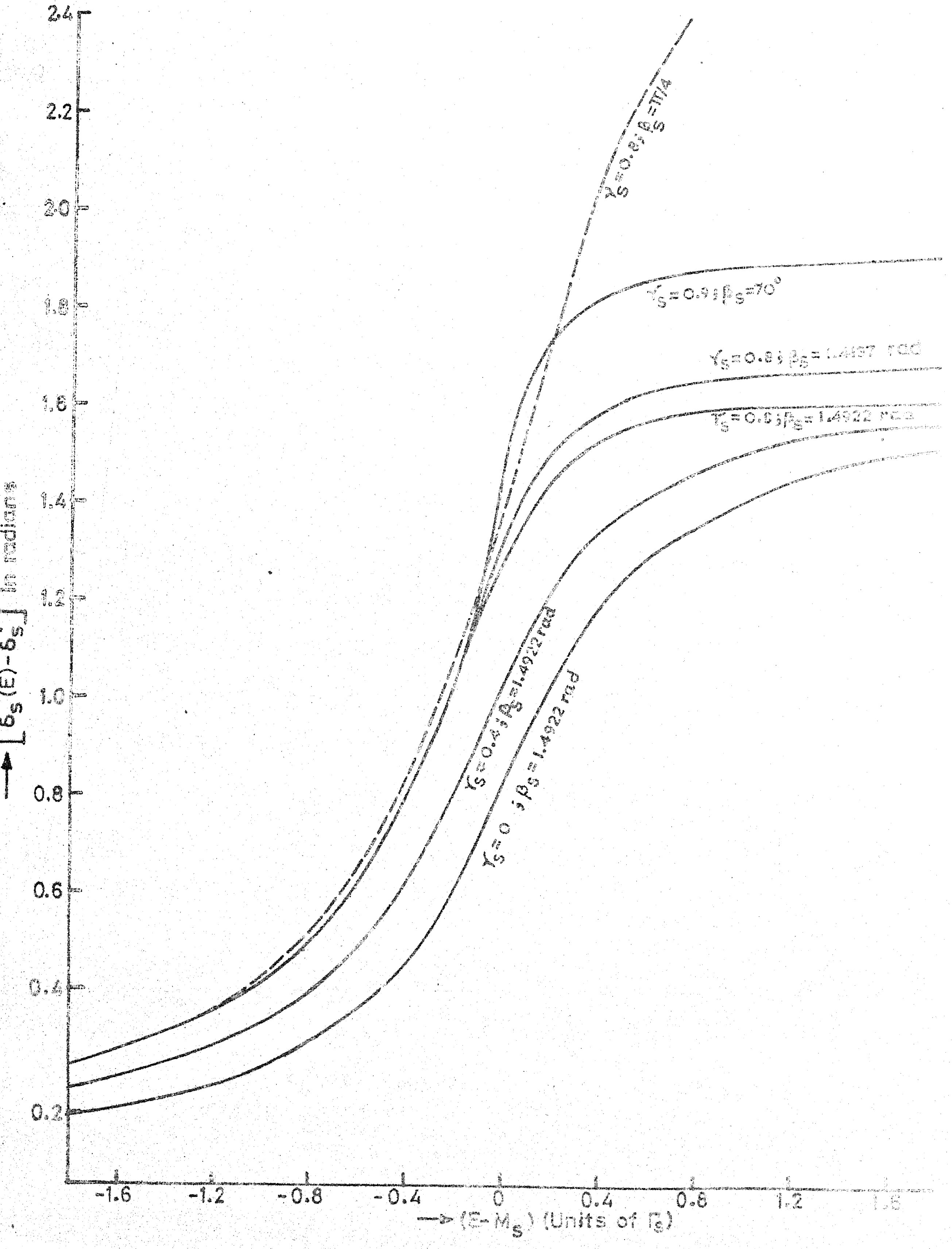
Since the delayed component is dominated by CP = -1 amplitudes the main branch of $\sigma(E)$ carrying the pole at m_L will be assigned the quantum number CP = -1. Similarly the main-branch of $\sigma(E)$ with a pole at m_S will be assigned CP = +1. Explicitely, we can write,

$$S_S(E) = \sigma_-^{(S)}(E) \quad \text{and} \quad \delta_S^t = \beta_S/2$$

$$S_L(E) = \sigma_+^{(L)}(E) \quad \text{and} \quad \delta_L^t = \pi-\beta_L/2 \qquad (4)$$

and

$$S_-(E) = \sigma_-^{(L)}(E) \quad \text{and} \quad \delta_-^t = \beta_L/2$$



$\gamma_S = 0.8; \beta_S = \pi/4$
$\gamma_S = 0.9; \beta_S = 70°$
$\gamma_S = 0.8; \beta_S = 1.4137$ rad
$\gamma_S = 0.8; \beta_S = 1.4922$ rad
$\gamma_S = 0.4; \beta_S = 1.4922$ rad
$\gamma_S = 0\ ; \beta_S = 1.4922$ rad
$\rightarrow \left[\delta_S(E) - \delta_S^T\right]$ in radians
$\rightarrow (E - M_S)$ (Units of Γ_S)

The set of additional parameters (β_S, γ_S; β_L, γ_L) determine the positions of branch points relative to the respective poles for m_S and m_L resonances.

3.3 STAGNATION OF THE PHASESHIFT $\delta_S(E)$:

The eigenphase repulsion because of the presence of the branch point in the vicinity of the pole also leads to a stagnation of the main eigenphase. The extent of this stagnation depends upon the quantity ξ determining the repulsion band. For a smaller value of ξ, we expect the main eigenphase to stagnate rather quickly. The explicit E-dependence for $\delta_S(E)$ (which corresponds to main eigenphase for pole at m_S) is shown in Figure 2. It is seen that for $\gamma_S \lesssim 1$ and $\beta \sim \pi/2$ (corresponding to $\theta_S \sim 0$ and ξ_S small) the phaseshift has nice stagnation.

The decomposition (II.9) used to construct the neutral kaon state in terms of the basis states $|L-\rangle$, $|S+\rangle$ and $|L+\rangle$ requires that the phaseshift $\delta_S(E)$ stagnates well before $E = m_L$. This will mean that the value of ξ_S should be much smaller than $\Delta m = m_L - m_S$. Explicitely, if $|\theta_S| \ll \Gamma_S$, then

$$\left|\frac{d\delta_S(E)}{dE}\right|^2 \sim \frac{\sin^2\beta_S}{4} \frac{[\Gamma_S/2(E-m_S) - \Gamma_S/2\sqrt{(E-m_S)^2+\Gamma_S^2/4}] + \xi_S^4}{[(E-m_S)^2 + \Gamma_S^2/4][(E-m_S)^2 + \xi_S^2]}$$

i.e. for $E \sim m_L$, $\delta_S(E)$ stagnates satisfactorily if

$$(\xi_S/\Gamma_S) \lesssim 1/4 \ .$$

The stagnation value, α_S, is given by

$$\alpha_S = (\pi - \beta_S) - x \tag{5}$$

where $(\pi - \beta_S)$ is the asymptotic value of $[\delta_S(E) - \delta_S^t(E)]$ for $E-m_S \gg \Gamma_S$ and the correction x is calculated using the form (1) and tabulated below.

Table I

Variation of $x = -\alpha_S + (\pi-\beta_S)$ for various values of ξ_S

(in degrees)

ξ_S/Γ_S / β_S (degrees)	1/20	1/10	3/20	1/5	1/4
90	0.5	1	1	1.5	1.5
85	1	2	2	4	5
80	2	2	4	5	7
75	3	3	5	8	11
70	4	5	7	10	15

3.4 TIME-DEPENDENCE OF THE AMPLITUDE:

The time-dependence of the state amplitude for K_o is given by

$$\langle K_o | K_o ; t \rangle = 4N^2 \int dE \, e^{-iEt} \sin^2 (\delta_- - \delta_-^t)$$

$$+ 4N^2 \int dE \, e^{-iEt} [\sin^2 (\delta_S - \delta_S^t)$$

$$+ e^{2i\alpha_S} \sin^2 (\delta_L - \delta_L^t)]$$

$$+ \text{transient term} \qquad (6)$$

and is characterized by integrals of the type,

$$I(t) = 2 \int dE \, e^{-iEt} \, \mathrm{Re} \, [e^{-2i\delta^t} \sigma(E) - 1] \qquad (7)$$

However, the expression (6) does not represent an experimentall measurable quantity because in any experiment with an unstable particle the time-dependent intensities are measured only for various partial - decays. The expression (6), nevertheless, can be related to these measured intensities by using Bell-Steinberger[25] relations (which are trivial identities in the present analysis because of in-built unitarity).

The time-dependent description of the decay in any analysis should follow faithfully the experimental situation. This will mean that for the time-localized experiments the final state should be represented by a wave packet with energy width $\Delta E \gg \Gamma$ (the width of the unstable state) about the mass of the decaying particle. This energy spread, ΔE, will be determined by the time-localization of the experiment.

Then, the observed time-dependent intensity for transition to a final state $|\alpha$, out $>$ for a resonance state, $|\psi_r>$ will be given by[26]

$$I_\alpha(t) = |\int dE e^{-iEt}\, b_\alpha(E) < E,\alpha,\text{out}\ \psi_r>|^2 / \int |b_\alpha(E)|^2\, dE$$

where $b_\alpha(E)$ has an energy spread $\Delta E >> \Gamma$ about $E = m_r$, and represents the particular act of measurement.

For the decay of neutral kaon [with a definition (II.5) for the resonance state] the intensity for decay into a channel with quantum-numbers collectively represented by m and CP-eigenvalue $\pm$ 1, therefore, will be,

$$I_{m,\pm}(t) = N^2 \left| \int dE\ e^{-iEt}\, [S_\pm(E)\ e^{-2i\delta^t_\pm} - 1]\ C^\pm_m \right|^2 \qquad (8)$$

The decay intensity is characterized by integrals of the type

$$J(t) = dE \int e^{-iEt}\, [\sigma(E)\ e^{-2i\delta^t} - 1] \qquad (9)$$

The integrals I(t) and J(t) can be evaluated explicitely with the form of the S-matrix eigenvalue functions given by (1). The detailed calculation shows that I(t) and J(t) are crucially dependent upon the position of the branch point. Since the case $\xi = 0$ is excluded, only the following cases are of interest.

(a) $\xi > \Gamma/2$:

When the branch point moves far away from both the real E-axis and the pole, the resonance is (almost completely) included in the branch $\sigma_-(E)$ and therefore, the corresponding phaseshift assumes a shape very near to the usual resonance shape. Consequently, the time-dependence of $I_-(t)$ and $J_-(t)$ is almost pure exponential while $I_+(t)$ and $J_+(t)$ are just the transient. If the branch point is near the pole (but $\xi > \Gamma/2$) then the leakage is very small and $I_+(t)$ and $J_+(t)$ exhibit non-exponential time-dependence of the form $e^{-\xi t}/\sqrt{t}$. The exponential term in $I_-(t)$ and $J_-(t)$ dominates over the non-exponential terms over a considerable time ($t \sim 10/\Gamma$).

(b) $0 < \xi < \Gamma/2$:

This is the most interesting case. Though none of the phaseshift resemble the usual resonance shape, there is, nevertheless, a sharp rise in the phaseshift which causes a time-delay in the associated amplitude. Therefore, the amplitudes $I(t)$ and $J(t)$ can be written as a sum of a composite part and a transient term (which becomes insignificant for $t \gtrsim 1/\Delta$, where $\Delta >> \Gamma$ is the cut-off implied in the integrations). The explicit calculation is presented in Appendix A. The main results are:

(i) The composite parts, denoted by $I^o(t)$ and $J^o(t)$, can be written as a sum of two terms, viz., (only for $0<|\theta|\lesssim\xi$),

$$I^o_{\pm}(t) = E_{\pm}(t) + f^{(1)}_{\pm}(t)$$

$$J^o_{\pm}(t) = E_{\pm}(t) + f^{(2)}_{\pm}(t)$$

where,

$$E_{\pm}(t) = (-i\pi\Gamma)(\gamma \sin\beta - i\cos\beta)\, e^{-imt-\Gamma t/2}\, e^{\pm i\beta} \tag{10}$$

represents the usual exponential term. It is interesting to note that both I_- [or $J_-(t)$] and $I_+(t)$ [or $J_+(t)$] contribute the exponential term with same width and mass. Therefore, the equality of the masses and width of the 'main'- and the 'leakage'- resonances is assured.

The non-exponential terms, $f^{(1)}$ and $f^{(2)}$ are given by,

$$\begin{aligned} f^{(1)}_{\pm}(t) = \sin\beta\, e^{-i(\theta+m)t} \Big[& e^{\pm i\beta}(\omega^2-\xi^2)e^{-\omega t}\int_\sigma^t dk e^{\omega k} K_o(\xi k) \\ & - e^{\mp i\beta}(\omega^{*2}-\xi^2)e^{+\omega^* t}\int_\sigma^t dk e^{-\omega^* k} K_o(\xi k) \\ & \mp 2i \sin\beta \left\{ \xi K_1(\xi t) - \frac{e^{-\xi t}}{t} \right\} \\ & - 2K_o(\xi t)\, \mathrm{Re}\,(e^{\pm i\beta}\omega) \Big] \end{aligned} \tag{11a}$$

and,

$$f_{\pm}^{(2)}(t) = 2\sin\beta\, e^{-i(\theta+m)t}\, e^{\pm i\beta}[(\omega^2-\xi^2)e^{-\omega t}\int_0^t dk\, e^{\omega k}K_o(\xi k)$$

$$- \omega K_o(\xi t) - \xi K_1(\xi t) + \frac{e^{-\xi t}}{t}] \qquad (11b)$$

where,

$$\omega = \Gamma/2 - i\theta.$$

(ii) For $\xi \sim \Gamma/4$, the $f^{(1)}$ and $f^{(2)}$ term also contribute an exponential term of the form,

$$Z_{\pm}(t) = (-i\pi\Gamma)(\gamma\sin\beta - i\cot\beta)\, e^{-imt-\Gamma T/2}$$

$$[-\frac{i}{\pi}\ln\frac{\sqrt{\omega'^2-\xi^2}+\omega'}{\omega'}]\, e^{\pm i\beta} \qquad (12)$$

where,

$$\omega' = \Gamma/2 - i|\theta|$$

Therefore, for this range of the parameters, we can write

$$I_{\pm}^{o}(t) = [E_{\pm}(t) \pm Z_{\pm}(t)] \pm h_{\pm}^{(1)}(t)$$

$$J_{\pm}^{o}(t) = [E_{\pm}(t) \pm Z_{\pm}(t)] \pm h_{\pm}^{(2)}(t)$$

where

$$h^{(j)}(t) = f^{(j)}(t) - Z^{(j)}(t)\ ; \quad j = 1,2 \qquad (13)$$

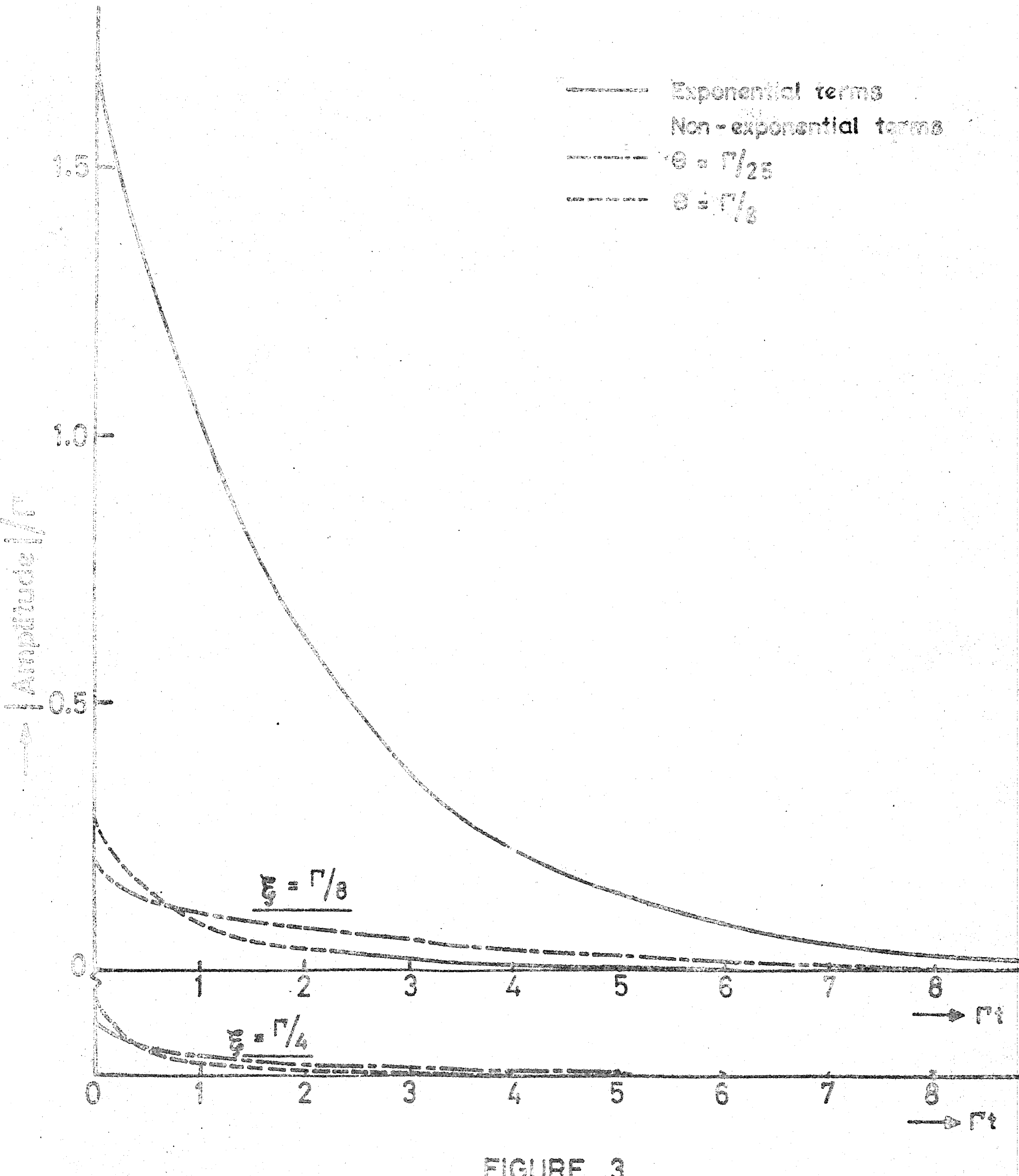

Exponential terms
Non-exponential terms
$\theta = \Gamma/25$
$\theta = \Gamma/3$
1.5
1.0
0.5
0
Amplitude $|/\Gamma$
$\xi = \Gamma/8$
1
2
3
4
5
6
7
8
Γt
$\xi = \Gamma/4$
0
1
2
3
4
5
6
7
8
Γt

FIGURE 3

It is interesting to note that both $I(t)$ and $J(t)$ have same exponential time-dependence. This, of course, is expected because the decay amplitudes for partial decays i.e. $J(t)$ and the total amplitude for the remaining undecayed kaons i.e. $I(t)$ are related by simple B-S relations[25].

The non-exponential terms $h(t)$ are computed numerically. A typical non-exponential form, e.g. the function $|h_-^{(2)}(t)\operatorname{cosec}\beta|$, is compared with the exponential form, $|E_-(t)|$, in Figure 3. It is seen that the non-exponential terms are very prominent for smaller values of ξ and θ. This is obvious intutively also because a smaller value of ξ and θ will mean a larger leakage and a larger deficiency in reaching the normal resonance structure for the eigen phases. For larger value of ξ ($\sim\Gamma/2$), though the non-exponential terms in main amplitude are small, the magnitude of leakage is very small and the leakage amplitude is non-exponential. In such cases, as noticed in the last section, the main eigenphase does not stagnate well enough so that the approximation (II.9) could be carried. The result of the computation are summarized in Table II.

Though the exponential decay has been experimentally varified to quite a degree of accuracy in nuclear reactions, the experimental data for particle decays is far from adequate. For example, in the measurement of the life-time of the delayed

kaon[27], the exponential decay law is verified with in 5% only for a time $\lesssim 3/\Gamma_L$. Beyond a few life-times the errors are so large that any satisfactory analysis of the data is very difficult.

Table II

The Comparison of Non-Exponential Terms with Exponential Terms

$r = | h_-^{(2)}(t)/E_-(t)| \operatorname{cosec} \beta$ in percent

Γt	$\theta = \Gamma/25$		$\theta = \Gamma/8$		$\theta = \Gamma/4$	
	$\xi = \Gamma/4$	$\xi = \Gamma/8$	$\xi = \Gamma/4$	$\xi = \Gamma/8$	$\xi = \Gamma/4$	$\xi = \Gamma/8$
0	5	12	7	15	10	25
1	4	9	2	8	2	15
2	4	7	2	7	1	10
3	3	5	1	6	1	8
5	3	3	1	4	1	8
10	7	25	5	10	3	8

Nevertheless, at least for the particles with longer life-times (e.g. pions, kaons etc.) one assumes that the exponential decay law is valid to a fair degree of accuracy (for a few life-times at least). That will mean, if we allow a considerable leakage of thepole, the position of the branch-

point must be such that

$$\xi \sim \Gamma/4 \quad \text{and} \quad |\theta| \gtrsim \Gamma/25$$

If we combine it with the requirement that the main eigen-phases are properly stagnating, we will have to fix $\xi \approx \Gamma/4$. The value of θ (though quite small) is left as an adjustable quantity. The amplitude will, then, be given by $[E_{\pm}(t) \pm Z_{\pm}(t)]$.

3.5 2π - INTERFERENCE AND POSITION OF BRANCH-POINT:

The intensity for 2π-decay of $|K_o\rangle$ will be, according to the expression (7) and the decomposition (II.9),

$$I_{2\pi}(t) = \frac{|C^{+}_{2\pi}|^2}{\sum_m |C^{+}_m|^{2\,2}} \cdot N^2 \Big| \int dE\, e^{-iEt}\,[S_S(E)e^{-2i\delta^t_S} - 1] + e^{2i\alpha_S} \int dE\, e^{-iEt}\,[S_L(E)\, e^{-2i\delta^t_L} - 1]\Big|^2 \tag{14}$$

The composite part of the decay-amplitudes in (14) are given by the exponential terms, and therefore, we can rewrite (14) as,

$$I_{2\pi}(t) \doteq \frac{|C^{+}_{2\pi}|^2}{\sum_m |C^{+}_m|^2}\, \Gamma_S \,\Big| e^{-\Gamma_S t/2 - i m_S t} + \eta\, e^{i m_L t - \Gamma_L t/2}\Big|^2 \tag{15}$$

where,

$$\eta = e^{2i\alpha_S} \frac{E_+^L(0) + Z_+^L(0)}{E_-^S(0) + Z_-^S(0)} \qquad (16)$$

[$E^{S,L}$ and $Z^{S,L}$ refer to the functions defined in expressions (10) and (12) with the corresponding parameters for the pole at m_S and m_L respectively].

If $\xi \simeq \Gamma/4$ and $|\theta|$ be very small, a first glance at (16) and the explicit form for $E_\pm(t)$ [it can be immediately seen that $|Z(t)|$ is much smaller than $|E(t)|$] lead to an approximate estimation of the phase of η in terms of the parameters β_S and β_L i.e.

$$\varphi_\eta \simeq 2\alpha_S + \beta_L + \beta_S$$

An asymptotic value of $\alpha_S \approx (\pi - \beta_S)$ gives,

$$\varphi_\eta \approx (\beta_L - \beta_S) \bmod 2\pi \, .$$

Therefore, the parameters can be fixed such that

$$\beta_L - \beta_S \simeq \pi/4 \qquad \text{and} \quad \xi \simeq \Gamma/4 \, .$$

The assumption (2) (with $0 < \gamma < 1$) leads to solutions

$$|\theta_S| / \Gamma_S = |\theta_L| / \Gamma_L$$

i.e. if $\gamma_S = \gamma_L$ then $\sin\beta_S = \sin\beta_L$. This combined with above values of the parameters immediately lead to

$$\beta_S \approx \pi/2 - \pi/8$$

$$\beta_L \approx \pi/2 + \pi/8 \tag{17}$$

and,

$$\gamma_L = \gamma_S \approx 0.96$$

Then the parameter η has the value of its phase,

$$\varphi_\eta = 2\alpha_S + \beta_L + \beta_S + \varphi_1 + \cot^{-1}\left[\frac{\cos\beta_L}{\gamma_L \sin\beta_L}\right]$$

$$- \cot^{-1}\left[\frac{\cos\beta_S}{\gamma_S \sin\beta_S}\right]$$

and,

$$\varphi_1 = \text{phase}\left[\frac{1 - \frac{i}{\pi}\ln\Omega_L}{1 + \frac{i}{\pi}\ln\Omega_S}\right]$$

$$\Omega_j = \ln\left[\sqrt{\omega_j'^2 - \xi^2} + \omega_j'\right]/\ \omega_j' \quad ; \quad j = L \text{ or } S$$

with a value $\varphi_\eta \approx 50^\circ$ which is consistent with the experimental value[10] $\varphi_{+-} \approx (46.6 \pm 2.5)^\circ$; $\varphi_{oo} = (49 \pm 13)^\circ$ and the magnitude is given by,

$$\eta \approx 1.2\ (\Gamma_L/\Gamma_S)$$

which also consistent with the experimental value[10] $\eta \approx (2.17 \pm 0.07) \times 10^{-3}$.

A comparison of expression (15) with the conventional expression[25] in terms of transition amplitudes i.e.

$$I_{2\pi}(t) = |< 2\pi|T|K_S> e^{-im_S t - \Gamma_S t/2} + e^{-im_L t - \Gamma_L t/2}|^2 \tag{18}$$

and the definition,

$$\eta = \frac{<2\pi|T|K_L>}{<2\pi|T|K_S>}$$

leads to identifications

$$< 2\pi|T|S+ > = \frac{C^+_{2\pi}}{[\Sigma_m |C^+_m|^2]^{1/2}} \sqrt{\Gamma_S}$$

and $\quad \varepsilon_1 < 2\pi|T|L+> = \eta <2\pi|T|S+>$.

These relations can be generalized to any final state $|f>$,

$$<f|T|S+> = C^+_f\sqrt{\Gamma_S}/[\Sigma_n|C^+_m|^2]^{\frac{1}{2}} \tag{19}$$

$$\varepsilon_1 <f|T|L+> = \eta <f|T|S+> \tag{19}$$

and similarly,

$$<f|T|L-> = C^-_f\sqrt{\Gamma_L}/[\Sigma_n|C^-_n|^2]^{\frac{1}{2}} \tag{20}$$

CHAPTER IV

REGENERATION

The regeneration of K_s in a beam of K_L when it passes through a nuclear material is an important process enabling us to study interference between K_s and K_L. This phenomena is quite incisive in selecting the valid theory. The analysis presented in this chapter differs from the conventional CP-violating analysis, because the leakage state, $|L+\rangle$, does not possess the strong-interaction. This will mean that any component of $|L+\rangle$ present in the initial beam of K_L will be transmitted largely unaffected.

The regeneration processes considered are (i) coherent transmission through the regenerator and (ii) the incoherent scatterings from individual nuclei. The inelastic effects have been shown to be negligible and therefore, we will not consider them.[28]

4.2 COHERENT-TRANSMISSION REGENERATION:

The coherently scattered amplitude is sharply peaked in forward direction.[29] The total change in the beam amplitude ψ (where ψ denotes amplitude for the set of states $|K_o\rangle$, $|\bar{K}_o\rangle$ and $|K'\rangle$) when it traverses a distance dx in regenerator is the sum of the changes - (i) by the time-evolution of the

amplitude in a corresponding proper-time:

$$d\tau = dx \; \frac{(1-v^2)^{\frac{1}{2}}}{v}$$

where v is velocity of the kaon in the material; and (ii) the effect of coherent nuclear scattering. The nuclear contribution for a mono-energetic kaon beam with momentum, k, is given by,

$$(d\Psi)_{nucl} = dx \; \frac{2\pi i N}{k} \begin{pmatrix} f(0) & 0 & 0 \\ 0 & \bar{f}(0) & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (1)$$

where N is the nuclear density and f(0) and $\bar{f}(0)$ are forward nuclear scattering amplitudes for K_o and $\bar{K}_o$ respectively.

The total change in the beam amplitude can be expressed by the set of coupled equations:

$$\left(\frac{d\psi}{d\tau} \right)_{med} = [m] \qquad (2)$$

where the 3 x 3 matrix, [m], determines the time-evolution of the states in the regenerator. Explicitly [m] is given as:

$$[m] = \begin{pmatrix} iM_L + c\,\Sigma f & c\,\Delta f & c\,\Sigma f.\varepsilon \\ c\,\Delta f & iM_S + c\,\Sigma f & c\,\Delta f.\varepsilon \\ c\,\Delta f.\varepsilon' & c\,\Sigma f.\varepsilon' & iM_L \end{pmatrix} \qquad (3)$$

where,

$$\varepsilon = \tfrac{1}{2}(\varepsilon_1/x_1 + \varepsilon_2/x_2)$$

$$M_j = m_j - \frac{i}{2}\ \Gamma_j \qquad j = L \text{ or } S$$

$$\Sigma f = f(0) + \bar{f}(0)$$

$$\Delta f = f(0) - \bar{f}(0)$$

$$c = \frac{-i\pi vN}{(1-v^2)^{\frac{1}{2}}\ k}$$

and ε_1, ε_2 and ε' are defined as the superposition coefficients for $|L+\rangle$ and $|S+\rangle$ in the expressions for K_o (or $\bar{K}_o$) and K' respectively viz.,

$$|K_o\rangle = \frac{1}{\sqrt{2}}\left[\,|L-\rangle + \frac{1}{x_1}\,|S+\rangle + \frac{\varepsilon_1}{x_1}\,|L+\rangle\,\right]$$

$$|\bar{K}_o\rangle = \frac{1}{\sqrt{2}}\left[\,|L-\rangle + \frac{\varepsilon_2}{x_2}\,|L+\rangle - \frac{1}{x_2}\,|S+\rangle\,\right] \qquad (4)$$

$$|K'\rangle \simeq |L+\rangle - \varepsilon'\,|S+\rangle$$

and $|\varepsilon_1| \sim |\varepsilon_2| \sim |\varepsilon'| \sim \sqrt{\Gamma_L/\Gamma_S}$.

The matrix $[m]$ has the eigenvalues

$$\lambda_1 \simeq iM_L + c\Sigma f - r_\Delta fc + O(|\varepsilon|^2)$$

$$\lambda_2 \simeq iM_S + c\Sigma f + r_\Delta fc + O(|\varepsilon|^2) \qquad (5)$$

$$\lambda_3 \simeq iM_L$$

where the regeneration parameter, r, is given by

$$r = (c_\Delta f / i_\Delta M) \qquad (6)$$

and for most of the regenerators in use, $|r| \lesssim 1/10$.

Let the beam of K_L (arising from either K_o or $\bar{K}_o$) incident on the regenerator be denoted by

$$|K_L\rangle \equiv |L-\rangle + \varepsilon\,|L+\rangle \qquad (7)$$

Then, since the amplitude for the state $|L+\rangle$ is transmitted largely unaffected, the regeneration takes place mainly due to $|L-\rangle$. The solution of equation (2) for the incident component $|L-\rangle$, in terms of the basis states $|L-\rangle$, $|S+\rangle$ and $|L+\rangle$, is given by,

$$|\varphi_{L-}\rangle = |L-\rangle + \rho_S\,|S+\rangle + \varepsilon\rho_L\,|L+\rangle \qquad (8)$$

where,

$$\rho_S = r[1 - e^{i\Delta Md}] \quad ; \quad \rho_L = \frac{\Delta f}{\Sigma f}[e^{(\lambda_1-\lambda_3)d} - 1] \qquad (9)$$

and

$$d = \frac{(1-v^2)^{\frac{1}{2}}}{v} D$$

where D is thickness of regenerator.

The emergent K_L beam will, therefore, be described by a state

$$|K_L\rangle_d = e^{-iM_L t}|L-\rangle + \rho_S . e^{-iM_S t}|S+\rangle + \varepsilon(\phi_{13} + \rho_L)e^{-iM_L t}|L+\rangle \tag{10}$$

where,

$$\phi_{13} = \exp(\lambda_1 - \lambda_3)\, d$$

determines the relative amplitude of transmitted $|L+\rangle$ component with respect to the transmitted $|L-\rangle$ component of the incident beam. The expression (10) may be contrasted with the conventional expression[25,29] where the $|L+\rangle$ term does not possess a thickness dependence.

The quantity $(\lambda_1 - \lambda_3)\, d$ can be calculated by using optical-model description of $K_L N$ interaction[30], which gives,

$$|\Delta f/\Sigma f| \lesssim 1/5 \qquad \text{for} \quad k \sim 3\text{GeV}$$

and Σf is almost pure imaginary. Let σ_T be the total elastic cross-section for $K_L N$ scattering, given by,

$$\sigma_T = \frac{2\pi}{k}\, \text{Im}\, \Sigma f$$

then we can write,

$$(\lambda_1 - \lambda_3)\, d \simeq \frac{N\sigma_T D}{2}[1 + i \tan \varphi'] \tag{11}$$

where, $\varphi' = \text{phase}\,(\Sigma f/i)$.

It is clear that the magnitude of the amplitude, ϕ_{13} is always greater than 1.

4.3 INCOHERENT SCATTERING AND MULTIPLE EFFECTS:

The kaon may also get diffracted by the individual nuclei in the regenerator. Such a scattering is coherent for the constituent nucleons because the target nucleus remains in the same internal state. The scattering from different nuclei are, of course, incoherent. The average angle of scattering by arguments of optics will be,

$$\sin \langle \theta \rangle \sim 1.22 \frac{\lambda}{2R}$$

where $\lambda = 2\pi\hbar/k$ is de Broglie wave-length of kaon and $R \sim 2.4\ A^{1/3}$ fm. the radius of nuclear mass number A i.e.

$$\sin \langle \theta \rangle \sim \frac{300}{kA^{1/3}} \sim 0.04 \quad \text{for } k \sim 2 \text{ GeV/c and } A \sim 60$$

This means, that the angular-distribution for the $K_L N$ scattering in such a case can be approximated by Gaussian distribution.[29,31]

For heavy regenerators e.g. copper, the incoherently scattered kaon can not be distinguished from the coherently scattered kaon (emerging at $\theta = 0$) even for $k \sim 1$ GeV/C. Therefore, the contribution of such incoherent scattering should be considered in any calculation because it may not be insignificant. On the otherhand, for lighter regenerators e.g. deuteron, the incoherent scattering (even for momenta as large as 10 GeV/C) can be distinguished from the usual coherent transmission and therefore, will not affect the calculations seriously.

[illegible]

A single incoherent scattering at a point x, and a corresponding proper time τ, by a thin slab of thickness dx, will give an intensity for decay into a final state $|f>$ [28,30],

$$\frac{dI_f(\theta,x)}{d\Omega} = N dx \left|\frac{f(\theta)+\bar{f}(\theta)}{2}\right|^2 \Big| e^{-iM_L t} <f|T|L->$$
$$+ e^{-M_S t}\left[\rho_S + \frac{\Delta f}{\Sigma f}\, e^{i\Delta M(d-\tau)}\right] <f|T|S+>$$
$$+ \eta e^{-iM_S t}\left[2e^{(\lambda_1-\lambda_3)(d-\tau)} - 1\right] \frac{\Delta f}{\Sigma f} <f|T|S+> \Big|^2 \qquad (12)$$

where we have used the definition (III.19)

$$\varepsilon_1 <f|T|L+> = \eta <f|T|S+> .$$

The second term in (12) represents the regenerated $|S+>$ by coherent transmission along the broken path (i.e. the path from $x_0 = 0$ to point x and from x to D) and the third term represents regeneration of $|S+>$ by incoherent scattering at the point x. The fourth term represents the combined regeneration of $|L+>$. The unusual structure of the last term (compare with the similar results of R.H. Good et al[31]) is solely due to the fact that the state $|L+>$ does not possess nuclear interaction.

For very thick regenerators, the kaon may get multiply scattered. These multiple scatterings are some times very important and can contribute as large as 25% of the total regeneration effect.[28] The single incoherent scattering expression (12) can be generalized to the case of n-successive scatterings (see Appendix B) at points $\{ x_i, \tau_i \}$ such that

$$x_{i-1} \leq x_i \leq D \quad ; \qquad x_o = 0.$$

The general expression is, for $n \geq 1$,

$$I_n^f(t) = \int_0^D dx_1 \int_{x_1}^D dx_2 \ldots \int_{x_{n-1}}^D dx_n \, (N\sigma_D)^n \, R_n^f(t) \tag{13a}$$

where,

$$\begin{aligned} R_n^f(t) = \Bigg| & e^{-iM_L t} \langle f|T|L-\rangle + \langle f|T|S+\rangle e^{-iM_S t} \\ & x \left[\rho_S + \sum_{m=1}^{n} \frac{\Delta f}{\Sigma f} e^{i\Delta M(d-\tau_m)} \right] + \eta \langle f|T|S+\rangle \\ & x \, e^{-M_L t} \frac{\Delta f}{\Sigma f} \left[2e^{(\lambda_1 - \lambda_3)(d-\tau_n)} - 1 \right] \Bigg|^2 \end{aligned} \tag{13b}$$

The occurrence of only τ_n in the last term is the consequence of the unusual characteristic of $|L+\rangle$ (the contribution to the intensity coming only from the transmission from the last scattering point to the exit). The diffraction cross-section, σ_D can be calculated in the optical-model approximation.[30]

The total intensity for the decay into the final state $|f\rangle$ is obtained by summing over all scattering orders, taking into account the detection efficiency, $e^f_{(n)}$ of the experiment for each scattering order. The total intensity, then, is given by,

$$I^f(t) = \sum_{n=0}^{\infty} I^f_n(t) \; e^f_{(n)} \tag{14}$$

where $I^f_o(t)$ is the intensity for coherent transmission regeneration and $e^f_{(o)} = 1$.

By an inspection of expression (13b) and (14), it is immediately obvious that the total contribution of $|L+\rangle$ in the emerging K_L beam following regeneration, relative to the unscattered $|L-\rangle$ is reduced by a factor,

$$G^f = \sum_{n=0}^{\infty} (N\sigma_D \, D)^n \; \frac{e^f_{(n)}}{n!} \tag{15}$$

where we have used the integration,

$$\int_0^D dx_1 \int_{x_1}^D dx_2 \cdots \int_{x_{n-1}}^D dx_n = \frac{D^n}{n!} \; .$$

Since for most of the detectors $e_{(n)} \approx (e)^n$ where e is the average detection efficiency, $G \approx \exp\,(N\sigma_D\, De)$ and is as large as 2.2 for copper regenerator of thickness 40 cms. This will mean that the rather strong thickness dependence occurring for thin regenerators [only coherent transmission, expression (10)] in the interference term involving $|L+\rangle$ is (after all) not so strong for thick regenerators with multiple scatterings.

CHAPTER V

2π-INTERFERENCE AND LEPTONIC CHARGE ASYMMETRY

In the last chapter, we considered a CP-preserving phenomenology for neutral kaon decay. In this chapter we will investigate the two important decay phenomena namely the 2π-interference and the leptonic charge-asymmetry, in the proposed analysis. The relevent quantities are calculated and compared with experiments. It is found that the existing experimental data is consistent with the proposed CPP-analysis. The calculations are also compared with those of the conventional CPV-analysis and certain distinctive features brought about clearly. Some possible tests to ascertain the fate of the present analysis are also suggested.

5.2 2π-INTERFERENCE:

The deviations from the Gellmann-Pais[1] scheme (which will mean an existence of CP = +1 delayed mode) are best observed in the decays of neutral kaon into 2π. Any component of CP = +1 delayed mode will lead to an interference phenomena in the decay of K_o or $\bar{K}_o$. The phenomenon is parameterized in terms of the amplitude ratio,

$$\eta_{2\pi} = \text{amp } (K_L \to 2\pi)/\text{amp } (K_S \to 2\pi) \tag{1}$$

Any experimental observation of the interference will involve an investigation of the time-dependent decay intensities. These decay intensities, in any phenomenological analysis, will be defined in Kerler's sense[26] [as discussed in Section (3.3)]. For example, the time-dependent 2π-decay intensity for decay of K_o in vacuum is given by,

$$I\,(K_o \to 2\pi) = |\langle 2\pi|T|S+\rangle|^2\,[e^{-\Gamma_S t} + |\eta|^2\, e^{-\Gamma_L t} + 2|\eta|\, e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m t - \varphi_\eta)] \tag{2}$$

where the complex quantity η is defined by the expression (II.16) in terms of the parameters (Θ,ξ) fixing the position of the branchpoint; and the transition amplitude $\langle 2\pi|T|S+\rangle$ is given as,

$$\langle 2\pi|T|S+\rangle = C^{+}_{2\pi}\sqrt{\Gamma_S}/[\sum_m |C^{+}_m|^2]^{\frac{1}{2}}$$

Similarly, the decay intensity for $\bar{K}_o$ will be given by,

$$I(\bar{K}_o \to 2\pi) = |\langle 2\pi|T|S+\rangle|^2\,[e^{-\Gamma_S t} + |\eta|^2 . |\varepsilon_2/\varepsilon_1|^2\, e^{-\Gamma_L t} - 2|\eta||\varepsilon_2/\varepsilon_1|\, e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m t - \varphi_\eta - \varphi')] \tag{3}$$

where, $\qquad \varphi' = \arg(\varepsilon_2/\varepsilon_1)$;

and ε_2 and ε_1 are the parameters introduced in Section (2.6)

The magnitude of the quantities ε_1 and ε_2 are of the same order i.e. $|\varepsilon_1| \sim |\varepsilon_2| \sim \sqrt{\Gamma_L/\Gamma_S}$.

We notice that the decay-intensities for K_o and $\bar{K}_o$ are different. This difference arises from the phenomenological definition of $|\bar{K}_o>$ [see Section 2.6(b)] and reflects the observation that the usual particle-antiparticle relations may not be valid for K_o and $\bar{K}_o$. The quantity,

$$D(t) \equiv [I(\bar{K}_o \to 2\pi) - I(K_o \to 2\pi)]\, e^{(\Gamma_S+\Gamma_L)t/2}/4|\eta| \qquad (4)$$

will be a measure of these deviations. In the present phenomenology, we expect,

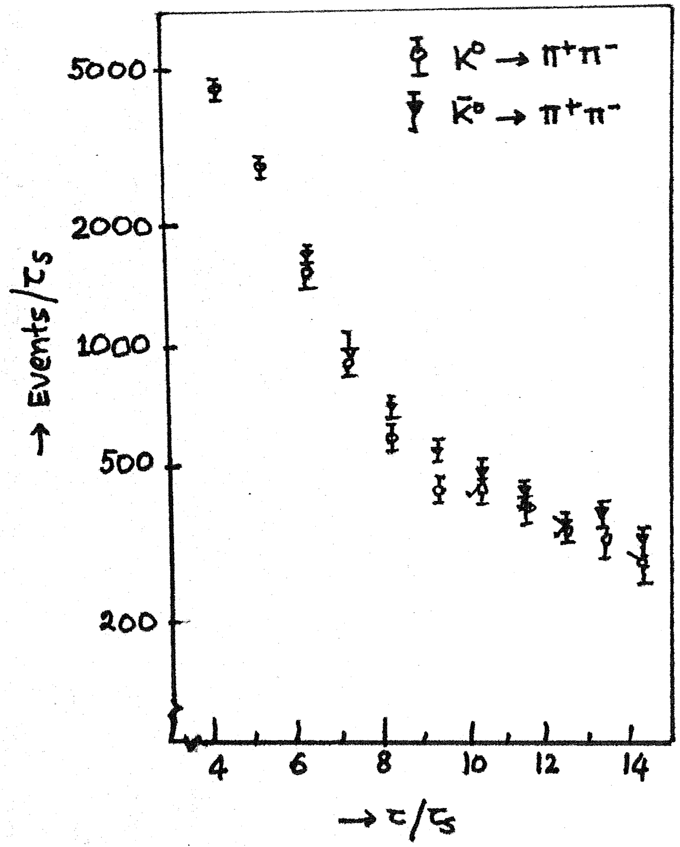
$$-D(t) = |\eta| \tfrac{1}{4}[1-|\varepsilon_2/\varepsilon_1|]e^{(\Gamma_S-\Gamma_L)t/2} + \tfrac{1}{2}[\cos(\Delta mt-\varphi_\eta) + |\varepsilon_2/\varepsilon_1|\cos(\Delta mt-\varphi_\eta-\varphi')] \qquad (5)$$

On the other hand, for the conventional CPV-phenomenology, where the 2π-decay-intensity is given by[25]

$$I_\pm(t) = |<2\pi|T|S+>|^2[e^{-\Gamma_S t}+|\eta|^2 e^{-\Gamma_L t} \pm 2|\eta|\, e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta mt - \varphi_\eta)] \qquad (6)$$

for the decay of K_o or $\bar{K}_o$ respectively, the difference, $D(t)$ is $-\cos(\Delta mt - \varphi_\eta)$[assuming preservation of CPT-symmetry]. Therefore, a comparison with Eq. (5) suggests that the two phenomenological results are identical if

$$\varepsilon_2 = \varepsilon_1 .$$


5000
2000
1000
500
200
→ Events/τ_s
$K^0 \to \pi^+\pi^-$
$\bar{K}^0 \to \pi^+\pi^-$
4
6
8
10
12
14
→ τ/τ_s
(a)

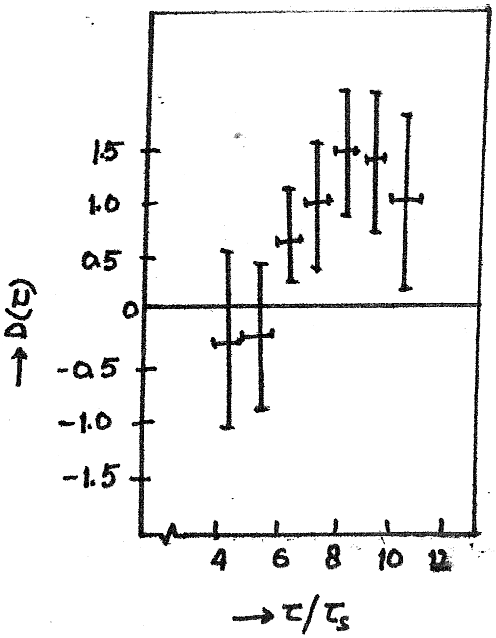

1.5
1.0
0.5
0
-0.5
-1.0
-1.5
→ $D(\tau)$
4
6
8
10
12
→ τ/τ_s
(b)

Figure 4

The latest experimental data(Banner et al[32])on $\pi^+\pi^-$ decay of pure K_o and pure $\bar{K}_o$ lead to a determination of $D(t)$. The best fit (Figure 4) is consistent with,

$$D(t) = -\cos(\Delta mt - \varphi_\eta)$$

which implies $\varepsilon_1 = \varepsilon_2$. That is, the $|\bar{K}_o>$ state is <u>not</u> the simple charge conjugation state (as usually is understood) of $|K_o>$. The deviations are of the order η.

(b) <u>Interference Following Regeneration</u>:

The intensity of 2π-decays following coherent-transmission regeneration in a nuclear material, has a time-dependence:

$$I_{2\pi}(t) = |<2\pi|T|S+>|^2 \, [|\rho_S|^2 e^{-\Gamma_S t} + |\phi_{13} + \rho_L|^2 e^{-\Gamma_L t}$$

$$+ 2|\rho_S||\eta| e^{-(\Gamma_S+\Gamma_L)t/2} \, \mathrm{Re}\{e^{i\Delta mt+i\varphi_s-i\varphi_\eta}$$

$$(\phi_{13}^* + \rho_L^*)\}] \qquad (7)$$

where, $\quad \phi_{13} \simeq \exp(N\sigma_T D/2)$

D is thickness of nuclear material, and $\rho_L \simeq (\phi_{13}-1)\Delta f/\Sigma f$.

Since $|\Delta f/\Sigma f| \lesssim 1/5$, the regenerated amplitude for $|L+>$ is much smaller than the transmitted amplitude, ϕ_{13}. Therefore $I_{2\pi}(t)$ exhibits a stronger dependence on the thickness of the regenerator compared to the CPV-analysis where the interference term has the thickness dependence arising from the ρ_S term <u>only</u>. Such a difference in the

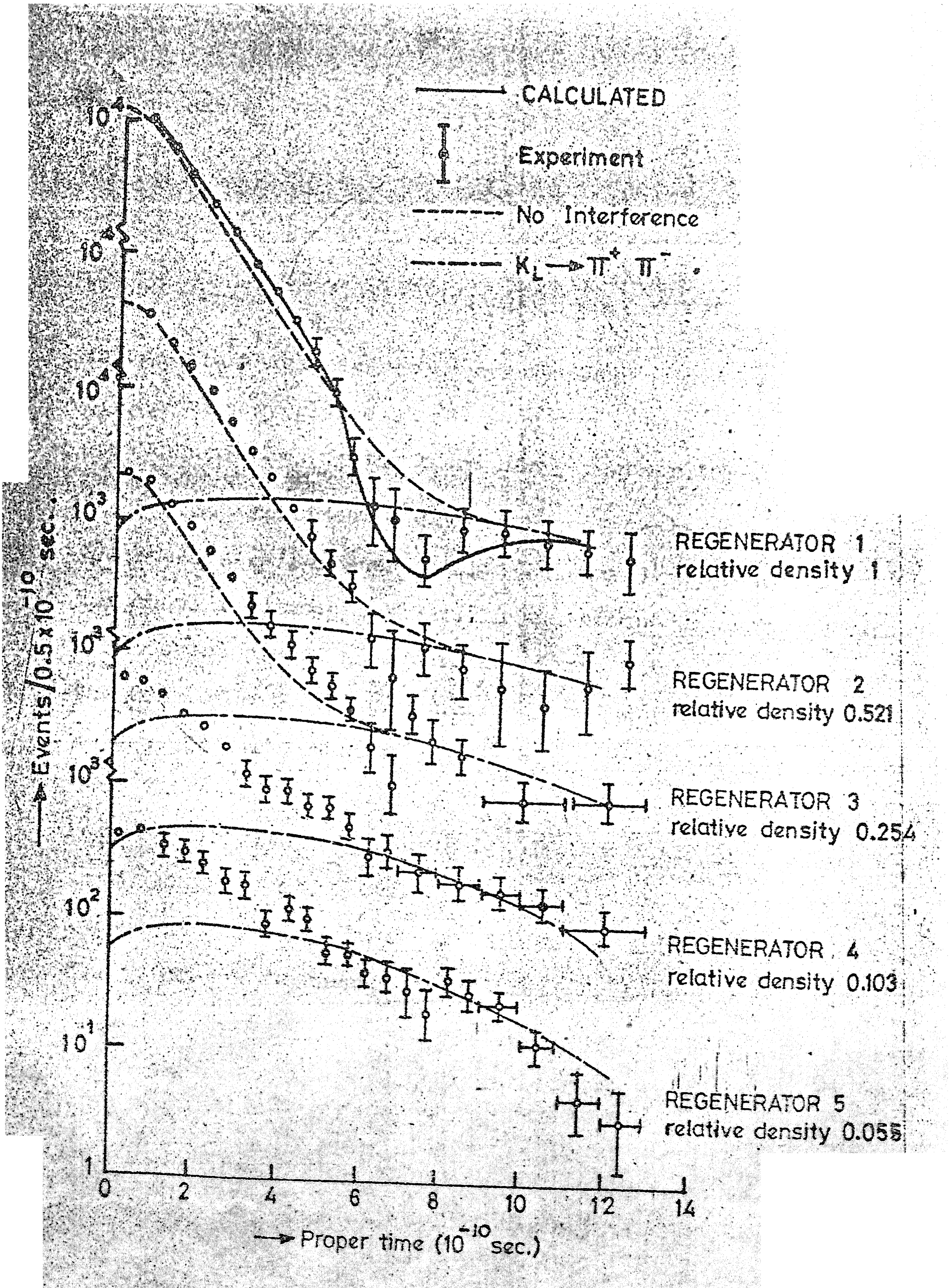
interference pattern is associated with the unusual behaviour of the state $|L+\rangle$. In the conventional CPV-analysis the state $|L+\rangle$ (which is nothing but $|L-\rangle$ itself) has the strong-interaction and therefore, is attenuated when subjected to nuclear interactions. On the otherhand, the $|L+\rangle$ state in the CPP-phenomenology does not possess any strong interaction and is therefore, transmitted largely unaffected. This will mean an enhancement of the amplitude for $|L+\rangle$ relative to the attenuated $|L-\rangle$ in the emerging beam of K_L. This enhancement, given by an amplitude ϕ_{13}, leads to the unusually strong interference.

For very thick regenerators, the multiple scattering in the regenerator become important. These multiple effects not only lead to a significant regeneration of $|L+\rangle$ but also an effective attenuation of the transmitted amplitude, ϕ_{13}, by the factor G which may be as large as 2. The corrected expression is,

$$
\begin{aligned}
I_{2\pi}(t) \simeq |\langle 2\pi|T|S+\rangle|^2 \Big[& |\rho|^2 e^{-\Gamma_S t} + |\eta|^2 |\phi_{13}|^2/G.\, e^{-\Gamma_L t} \\
& + 2|\eta|\,|\rho_S|/G.\, e^{-(\Gamma_S+\Gamma_L)t/2} . \mathrm{Re}\{ e^{i\Delta m t + i\varphi_\rho - i\varphi_\eta} \\
& [\phi_{13} + \rho_L\{1+(G-1)4\sigma_D/\alpha\sigma_T - F_1 G 4\sigma_D/\sigma_T\}]\} \Big]
\end{aligned}
\tag{8}
$$

where,

$$F_r G = \sum_{n=0}^{\infty} e^{(n+r)} \alpha^n/n! \; ; \; \alpha = N\sigma_D D$$



CALCULATED
Experiment
No Interference
$K_L \to \pi^+ \pi^-$
REGENERATOR 1
relative density 1
REGENERATOR 2
relative density 0.521
REGENERATOR 3
relative density 0.254
REGENERATOR 4
relative density 0.103
REGENERATOR 5
relative density 0.055
Events / 0.5×10^{-10} sec.
Proper time (10^{-10} sec.)

Figure.

and,

$$|\rho_1|^2 = |\rho_S|^2 \left[1+F_1(A-B)+EF_2\right]$$

where,

$$E = 4\sigma_D^2/\sigma_T^2 \quad ; \quad B \simeq 4\sigma_D/\sigma_T$$

$$A = \frac{E}{\alpha}\left|(1-e^{i\Delta Md})/\Delta Md\right|^{-2}\left[(1-e^{-\Gamma_S d})/\Gamma_S d\right]; \; \Delta M = \Delta m - \frac{i}{2}\Gamma_S$$

Though the thickness dependence is much more complicated than (7), a cursory glance at (8) suggests a less prominent thickness dependence for thick and heavy regenerators.

The experimental data with regenerators of different thicknesses by Faissner et al[33] (Figure 5) is consistent with the above results. However, in an attempt to seek for such enhancement of the interference, Darriulate et al[34] analysed the data of ref. 33 and concluded some what erronously an absence of such enhancements. The error in the analysis of Darriulate lies in ignoring the multiple effects. For the regenerator with a thickness D ≃ 25 cms of copper (as employed in the experiment of ref. 33), the multiple effects are quite important and contribute upto 25% (see Bannett[28]). Therefore, the thickness dependence as obtained by Darriulate should be compared with the corrected expression(taking into account the multiple effects) for decay-intensity following regeneration. Though no rigorous comparison can be made because of unavailability of raw data, the qualitative agreement is obvious.

5.3 LEPTONIC CHARGE ASYMMETRY:

The semi-leptonic decays of neutral K-meson is the only other phenomena besides the 2π-decays, where one can observe any interference effects. In $K^0_{l_3}$ decays, the interference in the decay amplitudes for $|L-\rangle$ and $|L+\rangle$ lead to a charge-asymmetry, defined by,

$$\delta = \frac{I(K_L \to \pi^- l^+ \nu) - I(K_L \to \pi^+ l^- \bar{\nu})}{I(K_L \to \pi^- l^+ \nu) + I(K_L \to \pi^+ l^- \bar{\nu})} \tag{9}$$

In general, δ is a function of the proper time because near the production point, there is always a small amount of K_S present which also contribute to the charge-asymmetry. The asymptotic value of the charge asymmetry is defined as the charge-asymmetry only in K_L (i.e. the K_S has decayed to insignificance). The charge-asymmetry following regeneration is an useful quantity because it determines the phase of the regeneration amplitude ; ρ_S for K_S.

The phenomena of semi-leptionic decay of neutral kaon can be described in terms of the CP-invariant amplitudes:

$$\begin{aligned} f &= \langle l^+\pi^-\nu|T|K_o\rangle = \langle l^-\pi^+\bar{\nu}|T|\bar{K}_o\rangle \\ g &= \langle l^-\pi^+\bar{\nu}|T|K_o\rangle = \langle l^+\pi^-\nu|T|\bar{K}_o\rangle \\ \text{and} \quad h &= \langle \pi^- l^+\nu|T|K'\rangle = \langle \pi^+ l^-\bar{\nu}|T|K'\rangle \end{aligned} \tag{10}$$

These amplitudes are real (by CPT-invariance). Further, the amplitude h can be expressed (approximately) in terms of

f and g, because the state $|K'>$ is mainly the leakage state $|L+>$ and by definition (III.17) we have,

$$\varepsilon < f|T|L+> = \eta < f|T|S+>$$

where, $\varepsilon = \varepsilon_1 = \varepsilon_2$.

This means,

$$\varepsilon h \simeq \eta(f-g) \tag{11}$$

Then, the charge-asymmetry arising from a state

$$|K,t> \equiv |L-> e^{-iM_L t} + \rho_S |S+> e^{-iM_S t} + \varepsilon\rho_L |L+> e^{-iM_L t} \tag{12}$$

is

$$\delta(t) \simeq \frac{(1-x^2)}{(1+x)^2} \left[2\mathrm{Re}\,(\eta\rho_L) + 2|\rho_S| e^{-(\Gamma_S-\Gamma_L)t/2} \cos(\Delta mt + \varphi_\rho)\right] \tag{13}$$

where $x = g/f$ determines the violation of $\Delta S = \Delta Q$ rule. The experiments[35] are consistent with $x \simeq 0$. The expression (13) can be studied in the following two case:

(a) Decay in Vacuum:

If the charge-asymmetry be measured without regeneration near the production point of K_o or $\bar{K}_o$, then, the charge asymmetry is (for sufficiently large $t \gtrsim 1/\Gamma_S$)

$$\delta(t) \simeq \frac{(1-x^2)}{(1+x)^2} \left[2\mathrm{Re}\eta \pm 2e^{-(\Gamma_S-\Gamma_L)t/2} \cos \Delta mt\right] \tag{14}$$

The asymptotic value of charge-asymmetry (i.e. for pure K_L beam) is

$$\delta_o = \frac{(1-x^2)}{(1+x)^2} \; 2 \,\mathrm{Re}\, \eta \qquad (15)$$

The time-dependence (14) is useful in the determination of Δm.[36]

(b) Decay Following Regeneration:

For thick regenerators such that $|\eta||\emptyset_{13}| << |\rho_S|$ the charge-asymmetry has the time-dependence

$$\delta(t) \simeq \frac{(1-x^2)}{(1+x)^2} \left[\delta_1 + 2|\rho_S| e^{-\Gamma_S t/2} \cos(\Delta mt + \varphi_\rho)\right]$$

where, $\delta_1 = 2 \,\mathrm{Re}\, (\eta \, \emptyset_{13})$.

The time-dependence of the charge-asymmetry following regeneration is identical to the CPV-results and determines the phase φ_ρ. However, the asymptotic value $(1-x^2)\delta_1/(1+x)^2$ is different from the asymptotic value δ_o (for decays in vacuum). This difference again, is associated with the unusual behaviour of $|L+>$ and therefore, will serve as a test for the existence of the state $|L+>$. For thick regenerators, the multiple scattering effects are also important and therefore, the corrected expression is,

$$\delta(t) \simeq \frac{(1-x^2)}{(1+x)^2} \Big[\delta_1' + 2|\rho_S| e^{-\Gamma_S t/2} \Big\{\cos(\Delta mt + \varphi_\rho) - \frac{2\sigma_D}{\sigma_T} F_1 \cos(\Delta mt + \varphi_\rho - \varphi'')\Big\}\Big] \qquad (16)$$

where, $\varphi'' = \arg(\Sigma f/i)$

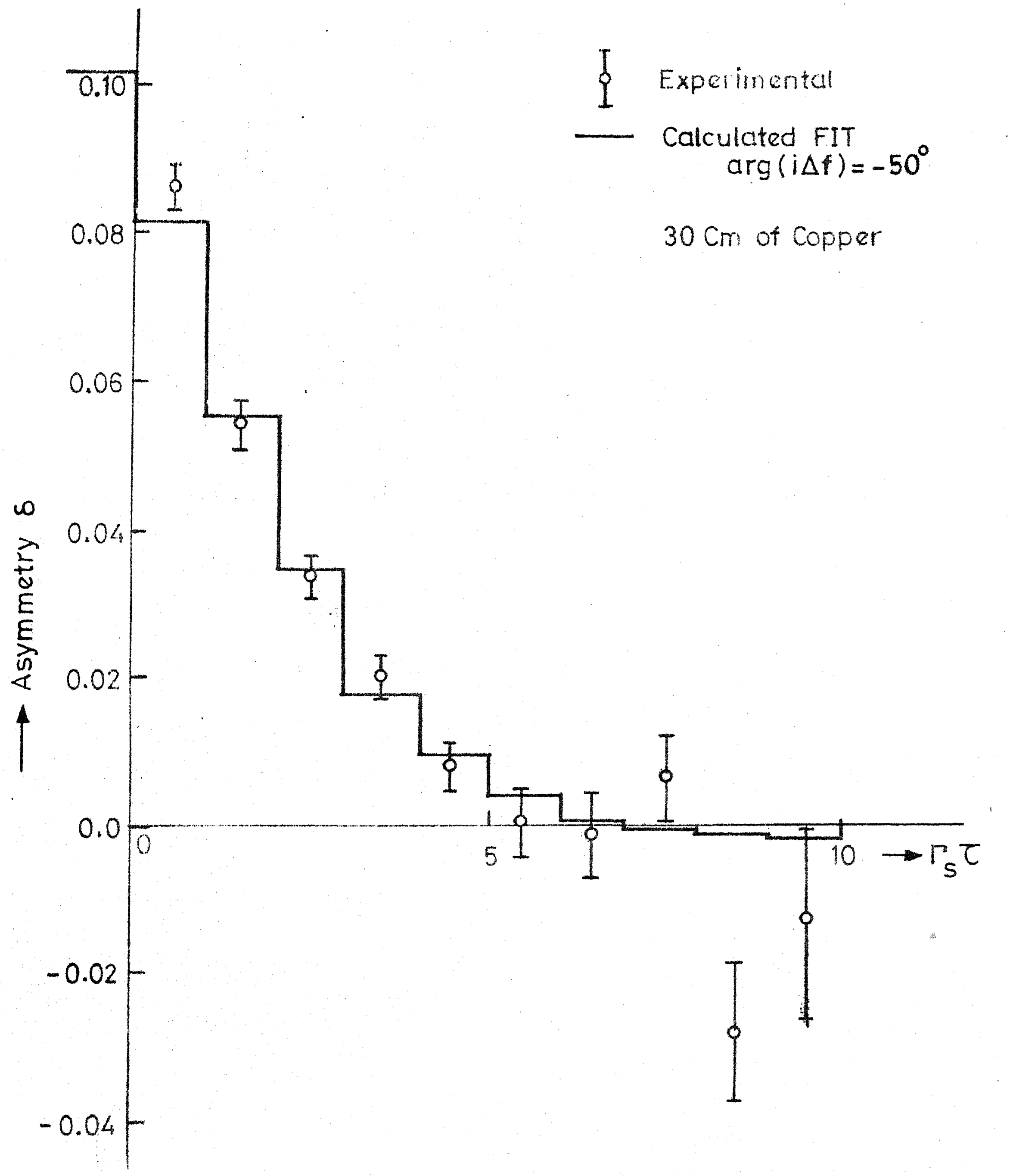

Experimental
Calculated FIT
arg(iΔf) = -50°
30 Cm of Copper
0.10
0.08
0.06
0.04
0.02
0.0
-0.02
-0.04
Asymmetry δ
0
5
10
$\Gamma_s \tau$

Figure.

and,

$$\delta_1' = 2\mathrm{Re}\ (\eta\emptyset_{13}/G) + 2\mathrm{Re}\ [\eta\rho_L(1+4\sigma_D/\alpha\sigma_T)(G-1)/G] \quad (17)$$

The time-dependence is again identical to the conventional expression[28] and agree with the experiments[28,37] (Figure 5). However, the determination of δ_1 by experiments is not at all simple. It can be seen from the expression (16) that even for moderately thick regenerators, the time-dependent term dominates the δ_1 ($\sim 1.5\ \delta_0$) even for time as large as $t \sim 10/\Gamma_S$; and as shown in Figure 5, the experimental errors are so large that no meaningful information about δ_1 can be extracted. For very thick regenerators, δ_1 could become larger or at least of the order of the time-dependent term so that it can be observed for small times. In such a situation, however, the overall beam intensity will be reduced so drastically that any experimental measurement will not be much reliable.

5.4 CONCLUDING REMARKS:

In the preceeding pages, a CP-preserving analysis of the neutral K-meson decay phenomena was presented. It was found that the phenomenological CP-preserving description of the kaon system necessitates inclusion of a pair of branch points at complex energies besides the usual pole to describe the decaying state in the S-matrix formalism. The occurrence of these branch points lead to a leakage

resonance-like behaviour in an orthogonal channel and consequently, the CP = +1 delayed component of K_L is interpreted as the leakage of the pole at m_L in CP = -1 channel.

Since the position of the branch point is determined completely by the weak-interaction responsible for the decay, the leakage state, $|L+\rangle$, does not possess any strong-interaction. Therefore, the relative amplitude of $|L+\rangle$ to that of $|L-\rangle$ is larger in a K_L beam following regeneration than without regeneration. Consequently, the decay-intensities following regeneration exhibit unusual dependence on thickness of the regenerator. The results can be contrasted with the results of CPV-phenomenology, where the same component K_L (which possesses strong interaction) leads to both the CP = +1 and CP = -1 decays.

All the experimental data to date[6,7,10,28,32-37] are seen to be consistent with the proposed CPP-analysis. The conclusive evidence against or in favour of the proposed analysis, however, can be obtained by performing certain experiments which distinguish the results of CPP-analysis from those of the usual CPV-analysis. Since the chief distinction between the two phenomenological approaches lies in the unusual behaviour of the leakage state $|L+\rangle$, any experimental verification of the proposed analysis will

involve a direct or indirect detection of the state $|L+\rangle$. The effects of the presence of $|L+\rangle$ are seen as:

(i) an enhancement of the two pion interference pattern for moderately thick regenerators over the pattern for very thin regenerators; and

(ii) the different time-independent (asymptotic) leptonic charge-asymmetry before and following the regeneration.

The study of first effect requires a detailed investigation of the decay intensities for various regenerator thicknesses, while the detection of the differences in asymptotic leptonic charge asymmetry will require a more accurate investigation of the $\delta(t)$ for moderately thick regenerators.

APPENDIX A

EVALUATION OF TIME-DEPENDENCE OF DECAY AMPLITUDE

The integrals characterizing the time-dependence of the state amplitude are,

$$I(t) \;=\; 2\int_{m-\Delta}^{m+\Delta} dE\; e^{-iEt}\; \mathrm{Re}\,[e^{-2i\delta^t}\,\sigma(E)-1] \qquad (A.1a)$$

and

$$J(t) \;=\; \int_{m-\Delta}^{m+\Delta} dE\;\; e^{-iEt}\;\; [e^{-2i\delta^t}\,\sigma(E)-1] \qquad (A.1b)$$

where the cut-off $\Delta >> \Gamma$ (the width of pole), and the background phases δ^t is such that $[\delta(E) - \delta^t]$ is negligible for $(E-M) < \sim \Gamma$. The S-matrix function is the multiple valued function,

$$\sigma_{\pm}(E) = \frac{(E-m)\cos\beta + \Gamma/2\gamma\sin\beta \pm i\sin\beta\,|E-E_c|}{E-m + i\Gamma/2}$$

where (β,γ) are the additional parameters, characterizing the position of branch points E_c and E_c^*, such that

$$E_c \equiv m + \theta + i\Gamma \qquad (A.2)$$

where, $\theta = \frac{\Gamma}{2}\gamma\cot\beta$ and $\xi = \frac{\Gamma}{2}\sqrt{1-\gamma^2}\,\mathrm{cosec}\,\beta$

The background phases, corresponding to $\sigma_{\pm}$, denoted by $\delta^t_{\pm}$, are given by $\pi-\beta/2$ and $\beta/2$ respectively.

The branch cuts extend from E_c to $-i\infty$ and E_c^* to $+i\infty$. The branch $\sigma_-(E)$ has a non-vanishing residue at the pole (for $\xi \neq 0$) given by $\Gamma[\gamma \sin\beta - i\cos\beta]$. This branch is designated as the main branch. The integrals (A.1a) and (A.1b) can be evaluated by contour integration over a contour C [Figure 1(a)] closed in the lower half of complex E-plane. Then, for $t > 0$ and $\xi > 0$, we can write,

$$I_\pm(t) = e^{-2i\delta_\pm^t} E(t)\,[1 \mp 1] \pm i\sin\beta\, e^{-imt}$$

$$[e^{-2i\delta_\pm^t} F(t, -\Gamma) - e^{2i\delta_\pm^t} F(t, +\Gamma)]$$

$$+ \Delta_\pm^1(t)\, e^{-imt} \qquad \text{(A.3a)}$$

$$J_\pm(t) = e^{-2i\delta_\pm^t} [E(t)\,(1 \mp 1) \pm 2i\sin\beta\, e^{-imt}$$

$$F(t, -\Gamma)] + \Delta_\pm^2(t)\, e^{-imt} \qquad \text{(A.3b)}$$

where,

$$E(t) = (-i\pi\Gamma)\, e^{-imt-\Gamma t/2} [\gamma \sin\beta - i\cos\beta] \qquad \text{(A.4)}$$

is the usual exponential term characteristic of a resonance, and

$$F(t, \pm\Gamma) = i\, e^{-i\theta t} \int_\xi^\infty dy\, e^{-yt} \left[\frac{\sqrt{y^2 - \xi^2}}{y + i\theta \pm \Gamma/2} - 1 \right] \qquad \text{(A.5)}$$

determines the discontinuity contribution across the branch cut. The transient terms are explicitely given by (for (for $t > 0$),

$$\Delta^{1}_{\pm}(t) \simeq -2\sin^2\beta\, e^{-\xi t-i\theta t-i\Delta t/2}\, \frac{\sin\Delta t/2}{t}$$

$$+ \gamma\Gamma \sin\beta\, C_i(\Delta t) \qquad (A.6a)$$

$$\Delta^{2}_{\pm}(t) \simeq \mp i \sin\beta\, e^{\mp i\beta}\, e^{-\xi t-i\theta t-i\Delta t/2}\, \frac{2\sin\Delta t/2}{t}$$

$$+ \left(-\frac{i\Gamma}{2}\right)(1 \mp \gamma\, e^{\mp i\beta})\, C_i(\Delta t) \qquad (A.6b)$$

where,

$$C_i(\Delta t) = \int_{\Delta t}^{\infty} \frac{\cos y}{y}\, dy$$

The discontinuity term (A.5) can be written as,

$$F(t,\omega) = ie^{-i\theta t}\left[\left(\frac{d^2}{dt^2} - \xi^2\right) g(t,\omega) - \frac{e^{-\xi t}}{t}\right] \qquad (A.7)$$

where,

$$g(t,\omega) = \int_0^{\infty} dx\, \frac{e^{-\xi t \cosh x}}{\xi\cosh x + \omega} \qquad (A.8)$$

and, $\omega = \mp\Gamma/2 + i\theta.$

For Re $\omega>0$, and $\xi>\Gamma/2$, $g(t,\omega)$ is given by,

$$g(t,\omega) = \int_0^{\infty} dy\, e^{-\omega y}\, K_0(\xi t + \xi y) \qquad (A.9a)$$

For $\xi<\Gamma/2$ and Re $\omega<0$, the integral (A.8) will contribute an exponential term, which can be separated out using the identity,

$$\text{Re}\ \omega<0: \quad \frac{e^{-xt}}{x+\omega} = \frac{e^{\omega t}}{x+\omega} - e^{\omega t}\int_0^t dk\, e^{-\omega k}\, e^{-xk} \qquad (A.10)$$

Then, (A.8) for $\mathrm{Re}\,\omega < 0$ and $\xi < \Gamma/2$ is given by,

$$g(t,\omega) = e^{\omega t} g(0,\omega) - e^{\omega t} \int_0^t dk\, e^{-\omega k} K_o(\xi k) \quad (A.9b)$$

where,

$$g(0,\omega) = i\pi/\sqrt{\omega^2-\xi^2} \quad \text{if } 0 < |\theta| \lesssim \xi.$$

Therefore, the 'leakage' amplitude $I_+(t)$ [and $J_+(t)$] has has the usual exponential term representing a resonance only if $0 < \xi < \Gamma/2$ and $0 < |\theta| \lesssim \xi$. The composite parts of $I(t)$ and $J(t)$ (denoted by $I^o(t)$ and $J^o(t)$ respectively), defined as,

$$\begin{aligned} I(t) &= I^o(t) + \Delta^1(t) \\ J(t) &= J^o(t) + \Delta^2(t) \end{aligned} \quad (A.11)$$

can be written as:

$$I^o_{\pm}(t) = e^{\pm i\beta} E(t) \pm \sin\beta\, e^{-imt-i\theta t} [e^{\pm i\beta} f_1(t) - e^{\mp i\beta} f_2(t)]$$

and,

$$J^o_{\pm}(t) = e^{\pm i\beta} [E(t) \pm 2 \sin\beta\, e^{-imt-i\theta t} f_1(t)]$$

where $f_1(t)$ and $f_2(t)$ include the non-exponential time-dependence. Explicitely,

$$f_1(t) = (\omega^2-\xi^2)\, e^{-\omega t} \int_0^t dk\, e^{\omega k} K_o(\xi k) - \omega K_o(\xi t) - \xi K_1(\xi t) + e^{-\xi t}/t \quad (A.12a)$$

$$f_2(t) = (\omega^{*2} - \xi^2)\, e^{\omega^* t} \int_\infty^t dk\, e^{-\omega^* k} K_0(\xi k)$$

$$+ \omega^* K_0(\xi t) - \xi K_1(\xi t) + e^{-\xi t}/t \qquad (A.12b)$$

where $\omega = \Gamma/2 - i\theta$.

The functions $f_1(t)$ and $f_2(t)$ can be computed numerically. However, for small $\xi \ll \Gamma/2$, the rather prominent non-exponential time-dependence of (A.12a) and (A.12b) is obvious. Even for small time ($\sim 1/\Gamma$) the non-exponential terms over take the exponential terms in the amplitudes $I^0(t)$ and $J^0(t)$. On the otherhand, for $\xi \sim \Gamma/2$, since the functions $K_0(\xi t)$ and $K_1(\xi t)$ can be approximated by their asymptotic expansions, even for $t \sim 1/\Gamma$, the non-exponential term which decreases faster than $e^{-\xi t}/t$ is small compared to the exponential term, $e^{-\Gamma t/2}$.

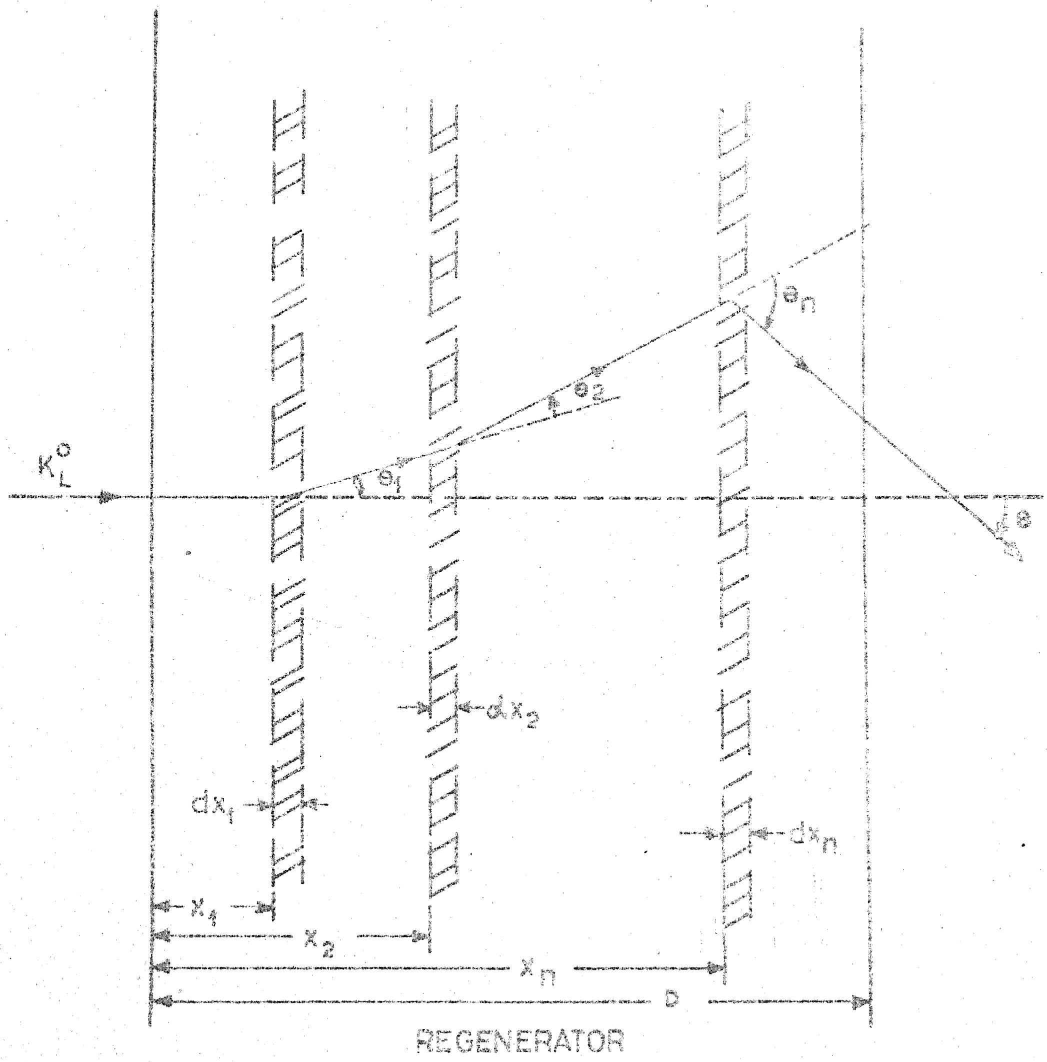
For intermediate values of ξ ($\sim\Gamma/4$), however, the time dependence requires a careful investigation, because in this region $f_1(t)$ may contribute a term with the time dependence of exponential type. $f_2(t)$, on the other hand, representing a discontinuity arising from $S^*(\Xi)$ will not (it is expected) contribute any such exponential t-dependence. The first term in (A.12) looks quite similar to the laplace transpormation of the function $K_0(\xi t)$ (at least for larger t). $f_1(t)$ gives an exponential terms of the form,

$$Z(t) = (-i\pi) \sqrt{\omega^2 - \xi^2}\; e^{-\xi t} \left(-\frac{i}{\pi} \ln \Omega\right)$$

where,

$$\Omega = [\sqrt{(\Gamma/2 - i|\theta|)^2 - \xi^2} + (\Gamma/2 - i|\theta|)]/(\Gamma/2 - i|\theta|).$$

The function $h(t) = f_1(t) - z(t)$ is computed numerically and compared to $E(t)$ in Figure 3 (Page 34). It is seen that the function $h(t)$ does not give a contribution more than ~10 % for $t \sim 5/\Gamma$.



K_L^o
θ_1
θ_2
θ_n
θ
dx_1
dx_2
dx_n
x_1
x_2
x_n
D
REGENERATOR

Figure. 7

APPENDIX B

MULTIPLE DIFFRACTION SCATTERING IN THEIR REGENERATOR

When a kaon passes through a thick regenerator, it may have n-successive scatterings at points corresponding to different depths and scattering angles. The scatterings from different depths and for different angles of scattering (even though they lead to same resultant angle θ) are incoherent. The presentation of this Appendix follows an earlier work of R.H. Good et al[31] except the corrections needed for the rather unusual state $| L+ \rangle$.

Let the n-successive scattering take place at depths $(x_1, x_2, \ldots, x_n)$ and with scattering angles $(\theta_1, \theta_2, \ldots, \theta_n)$ to give a resultant scattering angle θ (Figure 7). The total intensity will be obtained by integrating over all scattering angles and over all the depths such that

$$x_{i-1} \leq x_i \leq D \quad ; \quad x_0 = 0 \qquad \text{(B-1)}$$

$$i = 1, 2, \ldots, n$$

where D is the thickness of regenerator. Furthermore, the intensities for all scatter orders are added with the due weightage arising from various detection effeciencies of the apparatus. For n = 0, the process will correspond to the coherent-transmission regeneration.

Let $f(\theta)$ and $\bar{f}(\theta)$ be the scattering amplitudes for K_oN and $\bar{K}_oN$ respectively. Then, a single scattering will correspond to an ordinary scattering of $|S+\rangle$ and $|L-\rangle$ with an amplitude,

$$f_{22}(\theta) = [f(\theta) + \bar{f}(\theta)]/2,$$

and a regeneration of $|S+\rangle$ and $\varepsilon|L+\rangle$ by $|L-\rangle$ with an amplitude,

$$f_{21}(\theta) = [f(\theta) - \bar{f}(\theta)]/2.$$

The contribution at any point, (x_i, θ_i) can be calculated as follows:

(a) Ordinary Scattering:

The coherent-transmission of the state $|L-\rangle$ along the broken path (i.e. from the point x_{i-1} to x_i and from the point x_i to x_{i+1}) regenerates $|S+\rangle$ and $|L+\rangle$. The regenerated wave at point x_i is scattered. In this process, the state $|L+\rangle$ has a contribution only by transmission from x_i to x_{i+1} (in fact, to the exit surface of regenerator). The earlier (prior to point x_i) regeneration of $|L+\rangle$ corresponds to a lower-order scattering. That is, the single scattering contribution (relative to unscattered $|L-\rangle$) is,

$$f_{22}(\theta_1)\ [|L-\rangle + \rho_S\,|S+\rangle + \rho_L(x_1)\ |L+\rangle] \qquad \text{(B-2)}$$

where $\rho_L(x_1)$ is the regeneration amplitude from x_1 to D.

The generalization to the case of n-successive scattering is straight forward | L+ > contributing only at the last scattering point), and we write this contribution as,

$$F_n^1 = f_{22}(\theta_1) \ldots f_{22}(\theta_n) \left[|L-> + \eta_S |S+> + \varepsilon \eta_L(x_n) |L+> \right] \qquad \text{(B-3)}$$

(b) Regenerative Scattering:

At the point x_i, because $f(\theta) \neq \bar{f}(\theta)$ there will be an accumulation of the component of $|S+>$ and $|L+>$ preceded and followed by ordinary scattering. A single scattering will give a contribution:

$$f_{21}(\theta_1)\, e^{i\Delta M(d-\tau_1)}\, S+ + f_{21}(\theta_1)\, e^{(\lambda_1-\lambda_3)(d-\tau_1)}\, \varepsilon\, |L+> \qquad \text{(B-4)}$$

The generalization to n-successive scattering is again given by,

$$F_n^2 = \prod_{i=1}^{n} f_{22}(\theta_i) \left[\sum_{m=1}^{n} \left\{ \frac{f_{21}(\theta_m)}{f_{22}(\theta_m)} e^{i\Delta M(d-\tau_m)} |S+> + \frac{f_{21}(\theta_n)}{f_{22}(\theta_n)} e^{(\lambda_1-\lambda_3)(d-\tau_n)} \varepsilon |L+> \right\} \right] \qquad \text{(B-5)}$$

Since the angle of scattering is small, $\theta_i \sim 0.04$ rad., we can make the approximation,

$$\frac{f_{21}\ (\theta_i)}{f_{22}\ (\theta_i)} \simeq \frac{\Delta f}{\Sigma f}\ .$$

Therefore, the total contribution to order n is given by,

$$F_n = \prod_{i=1}^{n} f_{22}(\theta_i)\ [\,|L->+\{\rho_S + \frac{\Delta f}{\Sigma f} \sum_{r=1}^{n} e^{i\Delta M(d-\tau_r)}\}\ |S+>$$

$$+ \frac{\Delta f}{\Sigma f}\ \{2e^{(\lambda_1-\lambda_3)(d-\tau_n)} -1\}\ \varepsilon\,|L+>\ ] \qquad \text{(B-6)}$$

The expression (B-6) differs from the conventional expression[31] in the last term which is the contribution because of rather unusual behaviour of $|L+>$.

The angular integrations are carried over by assuming a Gaussian distribution for $|f_{22}(0)|^2$ for small angles i.e.

$$|f_{22}(\theta)|^2 \simeq |f_{22}(0)|^2\ e^{-\theta^2/2b^2} \qquad \text{(B-7)}$$

where b is the impact parameter such that the $K_L N$ diffraction cross-section is given by,

$$\sigma_D = 2\pi b^2\ |f_{22}(0)|^2. \qquad \text{(B-8)}$$

The n-Gaussian distributions are combined according to the law of combination,

$$\int \prod_{i=1}^{n} |f_{22}(\theta_i)|^2\ d\Omega_i = \sigma_D^n\ G_n(\theta) \qquad \text{(B-9)}$$

where $G_n(\theta)$ is the normalized Gaussian distribution for the resultant angle θ,

$$G_n(\theta) = \frac{e^{-\theta^2/2b^2}}{2\pi n b^2}$$

such that

$$\int G_n(\theta)\, d\Omega = 1.$$

The integration over depths is performed by Dyson's Technique[38]. Since the functions involved in the present case are simple functions, the symmetrization of the function is not necessary. That is,

$$\int_0^D dx_1 \int_x^D dx_2 \ldots \int_{x_{n-1}}^D f(D-x_i) = \frac{D^n}{n!} \int_0^D \frac{dx}{D} f(x) \tag{B-10}$$

Decay Intensities:

Since we are always interested in finding the time-dependent intensities for decays following regeneration, the intensity for appearance of any final state $|f\rangle$ is given by,

$n \geq 1$:

$$I_n^f(t) = (N\sigma_D)^n \int_0^D dx_1 \int_{x_1}^D dx_2 \ldots \int_{x_{n-1}}^D dx_n \; R_n^f(t) \tag{B-11}$$

where

$$R_n^f(t) = \Big| \langle f|T|L-\rangle e^{-iM_L t} + \langle f|T|S+\rangle e^{-iM_S t} \Big\{ \rho_S + \frac{\Delta f}{\Sigma f} \sum_{r=1}^{n} e^{i\Delta M(d-\tau_r)} \Big\} + \eta \langle f|T|S+\rangle e^{-iM_L t} \Big\{ 2e^{(\lambda_1-\lambda_3)(d-\tau_n)} - 1 \Big\} \Big|^2 \tag{B-12}$$

where the transition amplitudes $<f|T|L->$ and $<f|T|S+>$ are given in terms of the coefficients $C_f^{\pm}$, explicitely we have,

$$< f|T|L- > = C_f^- \sqrt{\Gamma_L}/ [\sum_n |C_n^-|^2]^{\frac{1}{2}}$$

and (B-13)

$$< f|T|S+ > = C_f^+ \sqrt{\Gamma_S}/ [\sum_n |C_n^+|^2]^{\frac{1}{2}}$$

Let $e^f_{(n)}$ denote the efficiency of detector for n-th order scattering, then the total resultant intensity for decay into final state $|f>$ is

$$I^f(t) = \sum_{n=0}^{\infty} I^f_n(t)\, e^f_{(n)}$$

where $I^f_o(t)$ is the intensity corresponding to the coherent-transmission regeneration i.e.

$$I^f_o(t) = \Big| <f|T|L-> e^{-iM_Lt} + <f|T|S+> \rho_S e^{-iM_St} + \eta <f|T|S+> e^{-iM_Lt} (\phi_{13} + \rho_L)\Big|^2$$

where $\phi_{13} = \exp(\lambda_1 - \lambda_3)\, d$ determines the amplitude of transmitted component of $|L+>$(present initially in the incident beam of K_L) relative to the transmitted component of $|L->$, and $e^f_o = 1$. In practice, $e_{(n)} \approx (e)^n$ where e is the average detection efficiency (≈ 0.9).

The two pion decay intensity is given by,

$$I_{2\pi}(t) = |\langle 2\pi|T|S+\rangle|^2 \Big[|\rho|^2 e^{-\Gamma_S t} + |\eta|^2 |\emptyset_{13}|^2/G\, e^{-\Gamma_L t}$$

$$+ 2|\eta|\,|\rho_S|/G\, e^{-(\Gamma_S+\Gamma_L)t/2}$$

$$\mathrm{Re}\{e^{i\Delta Mt+i\varphi_\rho-i\varphi_\eta}\,[\emptyset_{13} + \rho_L(1+\frac{4\sigma_D(G-1)}{\alpha\sigma_T}$$

$$- F_1 G\frac{4\sigma_D}{\sigma_T})]\}\Big]$$

where $\alpha = N\sigma_D D$; $G = \sum_{n=0}^{\infty} e_{(n)} \frac{\alpha^n}{n!}$; $G.F_r = \sum_{n=0}^{\infty} e_{(n+r)} \frac{\alpha^n}{n!}$

$$|\rho|^2 = |\rho_S|^2 [1 + F_1(A-B) + E F_2]$$

$$E = 4\sigma_D^2/\sigma_T^2 \quad ; \quad B \simeq 4\sigma_D/\sigma_T$$

$$A = \frac{E}{\alpha}\left|\frac{1-e^{i\Delta Md}}{\Delta Md}\right|^{-2} (1-e^{-\Gamma_S d})/\Gamma_S d$$

The leptonic charge asymmetry, defined by,

$$\delta(t) = \frac{I(K_L\to\pi^-l^+\nu) - I(K_L\to\pi^+l^-\bar{\nu})}{I(K_L\to\pi^-l^+\nu) + I(K_L\to\pi^+l^-\bar{\nu})}$$

can be written as,

$$\delta(t) \simeq \delta_1 + 2|\rho_S|\, e^{-(\Gamma_S-\Gamma_L)t/2} [\cos(\Delta mt+\varphi_\rho)$$

$$- \frac{2\sigma_D}{\sigma_T} F_1 \cos(\Delta mt + \varphi_\rho - \varphi'')]$$

for the thick xregenerators s.t. $|\rho_S| >> |\eta|$.

The asymptotic charge asymmetry, δ_1 is given by,

$$\delta_1 \;=\; 2\mathrm{Re}\,[\,\eta\,\phi_{13} + \eta\rho_L(1+4\sigma_D/\alpha\sigma_T).(G-1)]/G$$

and,

$$\varphi'' = \arg(\Sigma f/i)$$

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